Robust decentralized parameter identification for two-input two-output process from closed-loop step responses

Shao-Yuan Li[a,*], Wen-Jian Cai[b], Hua Mei[a], Qiang Xiong[b]

[a] Department of Automation, Institute of Automation, Shanghai Jiao Tong University, 1954 Hua-shan Rd, Shanghai 200030, PR China
[b] School of EEE, Nanyang Technological University, Singapore. 639798, Singapore

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Abstract

In this paper, a novel parameter identification method for closed-loop two-input two-output (TITO) processes from step-test is proposed. Through sequential step change of set points, the coupled closed-loop TITO system is decoupled equivalently into four independent single open-loop processes with same input signal acting on the four transfer functions. Consequently, existing identification methods for single-loop process can be extended to TITO systems and the parameters of first- or second-order plus dead-time models for each transfer function can be directly obtained by using the linear regression equations derived for the decoupled identification system. The proposed method is simple for engineering application and robust in the presence of large amounts of measurement noise. Simulation examples are given to show both effectiveness and practicality of the identification method for a wide range of multivariable processes.

Keywords: Identification; Two-input two-output process; Step test; Decoupled identification system; Least squares methods

1. Introduction

In multivariable process control, most schemes such as inverse *Nyquist* array or characteristic locus methods Astrom and Hagglund, 1984, 1995 require a full model of the process in the form of a transfer-function matrix or a frequency-response matrix over the entire working frequency range. In many cases, such a model is not available and physical modeling may require a prohibitive engineering effort. Therefore, practical and effective estimation of the full process models becomes appealing and has been an active research area of control engineering for a few decades. A considerable number of identification methods and their application to other engineering fields including advanced control strategy, optimization and signal processing have been reported in the literature (Loh & Vasnani, 1994; Poulin, Pomerleau, Desbiens, & Hodouin, 1996; Wang & Cai, 2003; Zhu & Butoyi, 2002).

There are two ways to identify a multivariable process for control application, i.e. open-loop and closed-loop ones. In any case, an excitation of the process is needed to extract useful information on process dynamics. For open-loop transient response experiments, step or pulse excitation signals are commonly injected at the process inputs, and the response is measured (Choi, Lee, Jung, & Lee, et al., 2000; Young, 1970). The main advantage of the step test is that the testing procedure is simple and requires little prior knowledge. However, it is quite sensitive to non-linearity in the system (Luyben, 1991). For closed-loop identification, a majority of existing techniques are in the frequency domain while the frequency range of interest for such applications is usually from zero up to the process critical frequency (Loh, Hang, Quek, & Vasnani, 1993; Shen, Wu & Yu, 1996; Wang & Shao, 1999; Wang & Cai, 2003). Since the closed-loop testing causes less perturbation to the process, it is preferred to open-loop one in process control practice.

*Corresponding author. Tel./fax.: +86-21-62932114.
E-mail address: syli@sjtu.edu.cn (S.-Y. Li).

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As the step test is the simplest and dominant in process control applications, there has been strong research interest in using such a test to determine the dynamics of unknown processes. Bi, Cai, Lee, and Wang, et al., 1999 proposed a simple yet robust identification method to obtain a first-order plus dead-time model for a linear monotonic process from a step test of open-loop control systems. The identification method was later applied to design auto-tuning PID controllers for heating, ventilation and air-conditioning (HVAC) systems (Bi, Cai, Wang, & Hang, et al., 2000), experimental results have demonstrated the effectiveness of the technique. Wang and Cluett 1994 presented an identification algorithm for processes operating in closed-loop, the algorithm involves fitting two *Laguerre* models directly to the control signal and the process output signal generated by a step change in the set-point. Wang, Guo and Zhang (2001); Wang and Zhang (2001) proposed some robust identification methods for linear time-delay processes from step responses in both time domain and frequency domain. These results was also applied to PID controller auto-tuning for multivariable processes Wang, Huang and Guo (2000).

In this paper, an engineering oriented identification technique for multivariable process is proposed, which extend SISO identification method in Bi, Cai, Lee, and Wang, et al., (1999) to the multivariable systems. The proposed method only requires step response data of closed-loop process, and no prior knowledge of the process dynamics and of the controller dynamics is needed. Through sequential step change of set points, the coupled closed-loop TITO system is decoupled equivalently into four individual single open-loop processes with same input signal acting on the four transfer functions. Consequently, existing identification methods for single-loop process can be extended to TITO systems and the parameters of first- or second-order plus dead-time models for each transfer function can be directly obtained by using the linear regression equations derived for the decoupled identification system. The proposed method is simple for engineering application and robust in the presence of large amounts of measurement noise. Various typical multivariable processes have been employed to illustrate the effectiveness of the method. It offers a good engineering tool for control engineers in retuning an existing multivariable control system and designing advanced controllers for multivariable processes.

2. Formulation of decentralized TITO Identification systems

Consider a TITO process under decentralized control as shown in Fig. 1, where r_i, e_i, u_i and y_i, $i,j = 1,2$ are set points, errors, controllers and process outputs, K_i, ζ_i and G_{ij} controllers, noises and process transfer functions, respectively. In general, K_1 and K_2 could be any type of controllers that make the closed loop system stable. To simplify our derivation, the notation r_i, e_i, y_i $i,j = 1,2$, are used in both s and t domain. The fundamental relationship between error signals and transfer function outputs for the system are expressed as:

$$\begin{aligned} y_1 &= G_{11}K_1e_1 + G_{12}K_2e_2, \\ y_2 &= G_{21}K_1e_1 + G_{22}K_2e_2, \\ e_1 &= r_1 - y_1, \\ e_2 &= r_2 - y_2. \end{aligned} \tag{1}$$

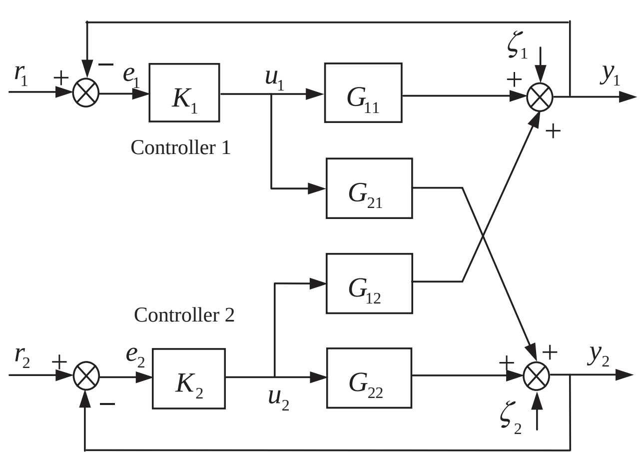

Fig. 1. Closed-loop TITO control system

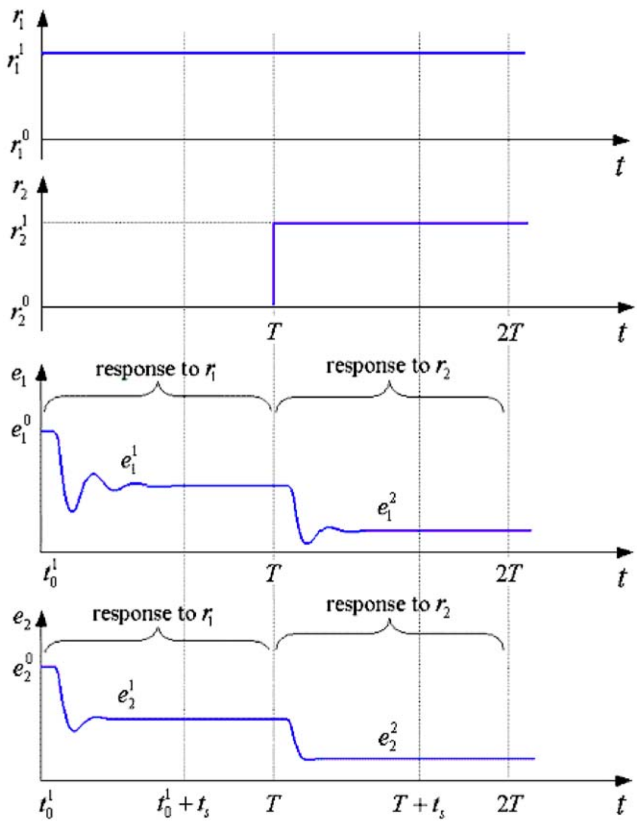

Fig. 2. The settling time t_s parameter identification.

Assume that the process initially rests at a steady state with initial set points, errors and output variables as $r_1^0, r_2^0, y_1^0, y_2^0, e_1^0$ and e_2^0, respectively, with

$$e_1^0 = r_1^0 - y_1^0,$$
$$e_2^0 = r_2^0 - y_2^0. \tag{2}$$

To identify the process parameters, the test involves the following two steps:

1. Make a step change in r_1 from r_1^0 to r_1^1, with r_2 kept unchanged, record the error signals for the two loops, until the first steady state is reached at $t \triangleq t_1 = T > t_s$ (see Fig. 2), t_s is the maximum settling time of all transfer functions. The incremental equation from initial state to the first state becomes:

$$\Delta y_1^1 = G_{11}K_1\Delta e_1^1 + G_{12}K_2\Delta e_2^1, \tag{3a}$$

$$\Delta y_2^1 = G_{21}K_1\Delta e_1^1 + G_{22}K_2\Delta e_2^1, \tag{3b}$$

where

$$\Delta e_1^1 = (r_1^1 - r_1^0) - (y_1^1 - y_1^0) = \Delta r_1 - \Delta y_1^1, \tag{3c}$$

$$\Delta e_2^1 = -(y_2^1 - y_2^0) = -\Delta y_2^1. \tag{3d}$$

2. Make a step change in r_2 from r_2^0 to r_2^1, while keeping $r_1 = r_1^1$ as before, record the error signals for the two loops, until the second steady state is reached at $t \triangleq t_2 = T > t_s$. Again, the incremental equation from the first state to the second state can be written as

$$\Delta y_1^2 = G_{11}K_1\Delta e_1^2 + G_{12}K_2\Delta e_2^2, \tag{4a}$$

$$\Delta y_2^2 = G_{21}K_1\Delta e_1^2 + G_{22}K_2\Delta e_2^2, \tag{4b}$$

where

$$\Delta e_1^2 = -(y_1^2 - y_1^1) = -\Delta y_1^2, \tag{4c}$$

$$\Delta e_2^2 = (r_2^2 - r_2^0) - (y_2^2 - y_2^1) = \Delta r_2 - \Delta y_2^2, \tag{4d}$$

Combine (3a), (3b), (4a) and (4b) into matrix form

$$Y = AX, \tag{5a}$$

where

$$Y \triangleq \begin{bmatrix} \Delta y_1^1 \\ \Delta y_2^1 \\ \Delta y_1^2 \\ \Delta y_2^2 \end{bmatrix}; A \triangleq \begin{bmatrix} K_1\Delta e_1^1 & K_2\Delta e_2^1 & 0 & 0 \\ 0 & 0 & K_1\Delta e_1^1 & K_2\Delta e_2^1 \\ K_1\Delta e_1^2 & K_2\Delta e_2^2 & 0 & 0 \\ 0 & 0 & K_1\Delta e_1^2 & K_2\Delta e_2^2 \end{bmatrix} \quad \text{and} \quad X \triangleq \begin{bmatrix} G_{11} \\ G_{12} \\ G_{21} \\ G_{22} \end{bmatrix}. \tag{5b}$$

Because of $\det(A) = -(K_1K_2)^2(\Delta e_1^1\Delta e_2^2 - \Delta e_1^2\Delta e_2^1)^2$, thus the matrix A is nonsingular if

$$\Delta e_1^1\Delta e_2^2 - \Delta e_1^2\Delta e_2^1 \neq 0.$$

From (3a–d) and (4a–d), it can be proved that

$$\Delta e_1^1\Delta e_2^2 - \Delta e_1^2\Delta e_2^1 = \det\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}\right) = \det(I + KG).$$

For a BIBO stable system, $\det(I + KG) \not\equiv 0$ $\forall s$ in *Laplace* domain. Therefore, A is usually nonsingular and X can be solved by

$$X = A^{-1}Y. \tag{6}$$

Substituting (3c–d), (4c–d) and (5b) into (6) with $\Delta u_i^j = K_i\Delta e_i^j$, $i,j = 1,2$, we obtain

$$\begin{aligned} G_{11}(s) &= \frac{\Delta y_1^1 \cdot \Delta u_2^2 - \Delta y_1^2 \cdot \Delta u_2^1}{\Delta u_1^1\Delta u_2^2 - \Delta u_1^2\Delta u_2^1} \triangleq \frac{y_{11}(s)}{u(s)}, \\ G_{12}(s) &= \frac{\Delta y_1^2 \cdot \Delta u_1^1 - \Delta y_1^1 \cdot \Delta u_1^2}{\Delta u_1^1\Delta u_2^2 - \Delta u_1^2\Delta u_2^1} \triangleq \frac{y_{21}(s)}{u(s)}, \\ G_{21}(s) &= \frac{\Delta y_2^1 \cdot \Delta u_2^2 - \Delta y_2^2 \cdot \Delta u_2^1}{\Delta u_1^1\Delta u_2^2 - \Delta u_1^2\Delta u_2^1} \triangleq \frac{y_{12}(s)}{u(s)}, \\ G_{22}(s) &= \frac{\Delta y_2^2 \cdot \Delta u_1^1 - \Delta y_2^1 \cdot \Delta u_1^2}{\Delta u_1^1\Delta u_2^2 - \Delta u_1^2\Delta u_2^1} \triangleq \frac{y_{22}(s)}{u(s)}. \end{aligned} \tag{7}$$

In terms of parameter identification, the coupled closed-loop TITO system has been decoupled into four individual single open-loop systems with same input signal acting on the four transfer functions.

Consequently, the problem of identification of coupled closed-loop TITO system is transformed into the identification of multi-single open-loop problem. The relation between the original system input/output and the decentralized identification system are given as

- The system input u in *Laplace* domain for all of the four loops is

  $$u(s) = \Delta u_1^1(s)\Delta u_2^2(s) - \Delta u_1^2(s)\Delta u_2^1(s) \tag{8}$$

  and that in time domain is

  $$u(t) = \Delta u_1^1(t) * \Delta u_2^2(t) - \Delta u_1^2(t) * \Delta u_2^1(t). \tag{9}$$

- Corresponding to the input u given in (8), the output signals y_{ij} both in *Laplace* domain and in time domain respectively for the equivalent identification system are

  1. $$\begin{aligned} y_{11}(s) &= \Delta y_1^1(s) \cdot \Delta u_2^2(s) - \Delta y_1^2(s) \cdot \Delta u_2^1(s), \\ y_{11}(t) &= L^{-1}[\Delta y_1^1(s) \cdot \Delta u_2^2(s) - \Delta y_1^2(s) \cdot \Delta u_2^1(s)] \\ &= \Delta y_1^1(t) * \Delta u_2^2(t) - \Delta y_1^2(t) * \Delta u_2^1(t). \end{aligned} \tag{10a}$$

2.
$$\begin{aligned} y_{21}(s) &= \Delta y_2^1(s)\cdot\Delta u_2^2(s) - \Delta y_2^2(s)\cdot\Delta u_2^1(s),\\ y_{21}(t) &= L^{-1}[\Delta y_2^1(s)\cdot\Delta u_2^2(s) - \Delta y_2^2(s)\cdot\Delta u_2^1(s)]\\ &= \Delta y_2^1(t)*\Delta u_2^2(t) - \Delta y_2^2(t)*\Delta u_2^1(t). \end{aligned} \tag{10b}$$

3.
$$\begin{aligned} y_{12}(s) &= \Delta y_1^2(s)\cdot\Delta u_1^1(s) - \Delta y_1^1(s)\cdot\Delta u_1^2(s),\\ y_{12}(t) &= L^{-1}[\Delta y_1^2(s)\cdot\Delta u_1^1(s) - \Delta y_1^1(s)\cdot\Delta u_1^2(s)]\\ &= \Delta y_1^2(t)*\Delta u_1^1(t) - \Delta y_1^1(t)*\Delta u_1^2(t). \end{aligned} \tag{10c}$$

4.
$$\begin{aligned} y_{22}(s) &= \Delta y_2^2(s)\cdot\Delta u_1^1(s) - \Delta y_2^1(s)\cdot\Delta u_1^2(s),\\ y_{22}(t) &= L^{-1}[\Delta y_2^2(s)\cdot\Delta u_1^1(s) - \Delta y_2^1(s)\cdot\Delta u_1^2(s)]\\ &= \Delta y_2^2(t)*\Delta u_1^1(t) - \Delta y_2^1(t)*\Delta u_1^2(t). \end{aligned} \tag{10d}$$

In such above formulas, the operator $*$ means a convolution operation.

Remark 1. *During the testing, step change for each set point is performed sequentially as shown in Fig. 2, where t_s is defined as the maximum time required for all errors to settle at steady state under the two testing.*

Remark 2. *By using incremental signals, zero initial conditions for the equivalent decentralized identification system are guaranteed, that is $u(t) = y_{ij}(t) = 0, t \leqslant 0$, for $i, j = 1, 2$, regardless the value of initial steady state errors.*

3. Least squares method

For the decentralized identification systems, the forward transfer function can be represented by

$$G_{ij}(s) = \frac{b_{ij1}s^{n-1} + b_{ij2}s^{n-2} + \cdots + b_{ijn}}{s^n + a_{ij1}s^{n-1} + a_{ij2}s^{n-2} + \cdots a_{ijn}}\,\mathrm{e}^{-L_{ij}s}, \quad i,j = 1,2. \tag{11}$$

Following Remark 2, initial conditions for the equivalent decentralized identification systems are zeros. Therefore, Eq. (11) can be written equivalently in differential equation form as

$$y_{ij}^{(n)}(t) + a_{ij1}y_{ij}^{(n-1)}(t) + \cdots + a_{ijn}y_{ij}(t) = b_{ij1}u^{(n-1)}(t - L_{ij}) + b_{ij2}u^{(n-2)}(t - L_{ij}) + \cdots + b_{ijn}u(t - L_{ij}). \tag{12}$$

Define

$$\int_{[0,t]}^{(m)} f(t) \triangleq \underbrace{\int_0^t \int_0^{\tau_m} \cdots \int_0^{\tau_2}}_{m} mf(\tau_1)\ \mathrm{d}\tau_1 \cdots \mathrm{d}\tau_m, \quad t \geqslant \max(t_1, t_2).$$

For an integer $m \geqslant 1$, Eq. (12) can be solved by integrating n times:

$$\begin{aligned} &y_{ij}(t) + a_{ij1}\int_{[0,t]}^{(1)} y_{ij}(t) + a_{ij2}\int_{[0,t]}^{(2)} y_{ij}(t) + \cdots + a_{ij(n-1)}\int_{[0,t]}^{(n-1)} y_{ij}(t) + a_{ijn}\int_{[0,t]}^{(n)} y_{ij}(t)\\ &\quad = b_{ij1}\int_{[0,t]}^{(1)} u(t - L_{ij}) + b_{ij2}\int_{[0,t]}^{(2)} u(t - L_{ij}) + \cdots + b_{ij(n-1)}\int_{[0,t]}^{(n-1)} u(t - L_{ij}) + b_{ijn}\int_{[0,t]}^{(n)} u(t - L_{ij}). \end{aligned} \tag{13}$$

Using first-order *Taylor* expansion for the unknown time delays, i.e., $\mathrm{e}^{-L_{ij}s} \doteq 1 - L_{ij}s$, Eq. (13) becomes

$$\begin{aligned} &y_{ij}(t) + a_{ij1}\int_{[0,t]}^{(1)} y_{ij}(t) + a_{ij2}\int_{[0,t]}^{(2)} y_{ij}(t) + \cdots + a_{ij(n-1)}\int_{[0,t]}^{(n-1)} y_{ij}(t) + a_{ijn}\int_{[0,t]}^{(n)} y_{ij}(t)\\ &\quad = b_{ij1}\int_{[0,t]}^{(1)} u(t) + b_{ij2}\int_{[0,t]}^{(2)} u(t) + \cdots + b_{ij(n-1)}\int_{[0,t]}^{(n-1)} u(t) + b_{ijn}\int_{[0,t]}^{(n)} u(t)\\ &\qquad - L_{ij}b_{ij1}u(t) - L_{ij}b_{ij2} - \cdots - L_{ij}b_{ij(n-1)}\int_{[0,t]}^{(n-2)} u(t) - L_{ij}b_{ijn}\int_{[0,t]}^{(n-1)} u(t). \end{aligned} \tag{14}$$

In process control applications, the first- or second- order plus time delay models are very popular since they can be used to represent both monotonic, oscillatory and non-minimum phase processes. The solutions for $n = 1$ and 2 under the framework of Eq. (14) are given as follows:

1. $n = 1$, Eq. (14) becomes

$$y_{ij}(t) = -a_{ij1}\int_0^t y_{ij}(\tau)\,\mathrm{d}\tau - L_{ij}b_{ij1}\int_0^t u(\tau)\,\mathrm{d}\tau + b_{ij1}u(t) \tag{15}$$

which can be written into the compact form

$$\gamma_{ij}^1(t) = [\phi_{ij}^1(t)]^{\mathrm{T}}\cdot\theta_{ij}^1, \tag{16}$$

where

$$\begin{aligned}\gamma_{ij}^1(t) &= y_{ij}(t),\\ [\phi_{ij}^1(t)]^{\mathrm{T}} &= \left[-\int_0^t y_{ij}(\tau)\ \mathrm{d}\tau \quad -\int_0^t u(\tau)\ \mathrm{d}\tau \quad u(t)\right],\\ \theta_{ij}^1 &= \left[a_{ij1} \quad L_{ij}b_{ij1} \quad b_{ij1}\right].\end{aligned} \tag{17}$$

2. $n = 2$, Eq. (14) becomes

$$\begin{aligned}y_{ij}(t) = &-a_{ij1}\int_0^t y_{ij}(\tau)\ \mathrm{d}\tau - a_{ij2}\int_0^t\int_0^\tau y_{ij}(\tau_1)\ \mathrm{d}\tau_1\ \mathrm{d}\tau + b_{ij1}\int_0^t u(\tau)\ \mathrm{d}\tau + b_{ij2}\int_0^t\int_0^\tau u_{ij}(\tau_1)\ \mathrm{d}\tau_1\ \mathrm{d}\tau\\ &- L_{ij}b_{ij1}u(t) - L_{ij}b_{ij2}\int_0^t u(\tau)\ \mathrm{d}\tau.\end{aligned} \tag{18}$$

It is again expressed as

$$\gamma_{ij}^2(t) = [\phi_{ij}^2(t)]^{\mathrm{T}}\theta_{ij}^2, \tag{19}$$

where

$$\begin{aligned}\gamma_{ij}^2(t) &= y_{ij}(t),\\ [\phi_{ij}^2(t)]^{\mathrm{T}} &= \left[-\int_0^t y_{ij}(\tau)\ \mathrm{d}\tau - \int_0^t\int_0^\tau y(\tau_1)\ \mathrm{d}\tau_1\ \mathrm{d}\tau \int_0^t u(\tau)\ \mathrm{d}\tau \int_0^t\int_0^\tau u(\tau_1)\ \mathrm{d}\tau_1\ \mathrm{d}\tau - u(t) \quad -\int_0^t u(\tau)\ \mathrm{d}\tau\right],\\ \theta_{ij}^2 &= \left[a_{ij1} \quad a_{ij2} \quad b_{ij1} \quad b_{ij2} \quad L_{ij}b_{ij1} \quad L_{ij}b_{ij2}\right].\end{aligned} \tag{20}$$

Eq. (15) and (18) can be solved by the least squares methods for each transfer functions $i,j = 1,2$, and $k = 1,2$ to form the regression form

$$\Gamma_{ij} = \Psi_{ij}\Theta_{ij}, \tag{21}$$

where $\Gamma_{ij} = [\gamma_{ij}^k(t_1),\ \ \gamma_{ij}^k(t_2),\ \ \cdots\ \ \gamma_{ij}^k(t_N)]^{\mathrm{T}}$, $\Psi_{ij} = [\phi_{ij}^k(t_1),\ \ \phi_{ij}^k(t_2),\ \ \cdots\ \ \phi_{ij}^k(t_N)]^{\mathrm{T}}$. Its least squares estimation for Θ_{ij} are

$$\tilde{\Theta}_{ij} = \left(\Psi_{ij}^{\mathrm{T}}\Psi_{ij}\right)^{-1}\Psi_{ij}^{\mathrm{T}}\Gamma_{ij}. \tag{22}$$

Once $\tilde{\Theta}_{ij}$ are found from Eq. (22), a_{ij1}, b_{ij1} and L_{ij} can be recovered from

$$\begin{bmatrix}a_{ij1}\\ b_{ij1}\\ L_{ij}\end{bmatrix} = \begin{bmatrix}\theta_{ij1}^1\\ \theta_{ij3}^1\\ \theta_{ij2}^1/\theta_{ij3}^1\end{bmatrix} \tag{23}$$

for $n = 1$, and a_{ij1}, a_{ij2}, b_{ij1}, b_{ij2} and L_{ij} from

$$\begin{bmatrix}a_{ij1}\\ a_{ij2}\\ b_{ij1}\\ b_{ij2}\\ L_{ij}\end{bmatrix} = \begin{bmatrix}\theta_{ij1}^2\\ \theta_{ij2}^2\\ \theta_{ij3}^2\\ \theta_{ij4}^2\\ \theta_{ij5}^2/\theta_{ij3}^2\end{bmatrix} \tag{24}$$

for $n = 2$, respectively.

Remark 3. *To integrate the equivalent input $u(t)$, the first-order Taylor series expansion is adopted for the time delay* i.e. $e^{-L_{ij}s} \doteq 1 - L_{ij}s$. *Even though it showed very accurate results in our simulation, this approximation may not be sufficient in some cases. More accurate L_{ij} can be obtained by either using higher-order Taylor series expansion or frequency domain identification approach.*

Remark 4. *In the noise-free testing environment, the least squares method* (22) *yields very good results. Now suppose that there exist white noises $\zeta_1$and ζ_2 as shown in* Fig. 1, then the equivalent input and outputs are rewritten as

$$\begin{aligned} u(t) &= \hat{u}(t) + \zeta_u, \\ y_{11}(t) &= \hat{y}_{11}(t) + \zeta_{y11}, \\ y_{12}(t) &= \hat{y}_{12}(t) + \zeta_{y12}, \\ y_{21}(t) &= \hat{y}_{21}(t) + \zeta_{y21}, \\ y_{22}(t) &= \hat{y}_{22}(t) + \zeta_{y22}. \end{aligned}$$

Here, $\hat{u}(t)$ and $\hat{y}_{ij}$, $i,j = 1,2$, are input/output signals; ζ_u, ζ_{yij} are colored noises, respectively, which are functions of ζ_1 and ζ_2. Due to the existence of these colored noises, $\tilde{\Theta}_{ij}$ will deviate from Θ_{ij}. A common method is to pre-treat $u(t)$ and $y_{ij}(t)$ through some low-pass filters. In the proposed method, we suggest integrating (14) in the both sides, which can eliminate stochastic noise effectively.

4. Simulation example

The Wood and Berry binary distillation column plant (Wang, Guo & Zhang 2001c) is a typical TITO process with strong interaction and significant time delays, which transfer function matrix is given as

$$G(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{1+16.7s} & \dfrac{-18.9e^{-3s}}{1+21s} \\ \dfrac{6.6e^{-7s}}{1+10.9s} & \dfrac{-19.4e^{-3s}}{1+14.4s} \end{bmatrix}.$$

For the decentralized closed-loop system with $K_{P1} = 0.5271$, $K_{I1} = 0.0763$, $K_{D1} = 0.45$, and $K_{P2} = -0.1064$, $K_{I2} = -0.018$ $K_{D2} = 0.02$, the step-test error signals without noise and the differentials of the equivalent signals u and y_{ij} $(i,j = 1,2)$ are shown in Figs. 3 and 4, respectively.

In practice, the real measurement of the process output under closed-loop test are inevitably corrupted by measurement noise, which leads to the corruption of the constructed test signals. To show the effectiveness of the proposed approach under measurement noise conditions, we define the noise-to-signal ratio as

$$\mathrm{NSR} = \frac{\mathrm{mean(abs(noise))}}{\mathrm{mean(abs(signal))}}. \tag{25}$$

In order to show the effectiveness of the proposed approach by simulations, a criterion in frequency domain is defined as

$$E = \max_{\omega \in [0,\omega_c]} \left\{ \left| \frac{\hat{G}(j\omega) - G(j\omega)}{G(j\omega)} \right| \times 100\% \right\}, \tag{26}$$

where $G(j\omega)$ and $\hat{G}(j\omega)$ are the actual and estimated process frequency response, respectively, and $\angle G(j\omega_c) = -\pi$.

The identified FOPDT models and parameters under difference noise level are given in Table 1. Fig. 5 shows the *Nyquist* curves of identified models under different NSR.

5. Application to multistage gas–liquid absorption column system

Consider a multistage gas–liquid absorption column shown in schematic form in Fig. 6 Giovani, Serkan and Oscar, 2003. The column features n trays where the transfer of a component of interest from the gas stream into the liquid stream is accomplished in each tray as the rising gas stream bubbles through a liquid layer that flows horizontally across the tray. The variable x and y, respectively, denote the composition of the species of interest in the liquid and gas phases. The manipulated variables are the compositions x_f and y_f of the feed streams, and the measured process variables are the compositions x_p and y_p of the product streams.

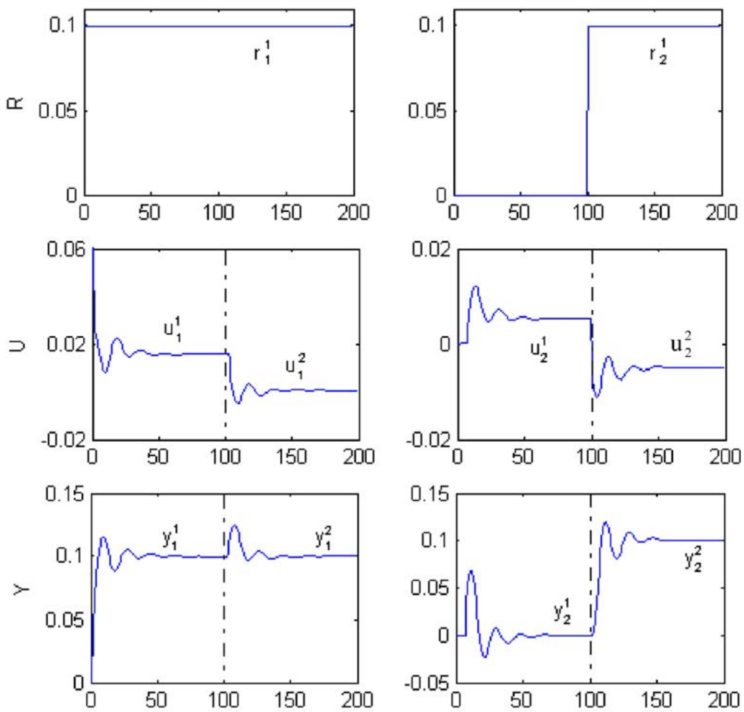

Fig. 3. System step tests without noises.

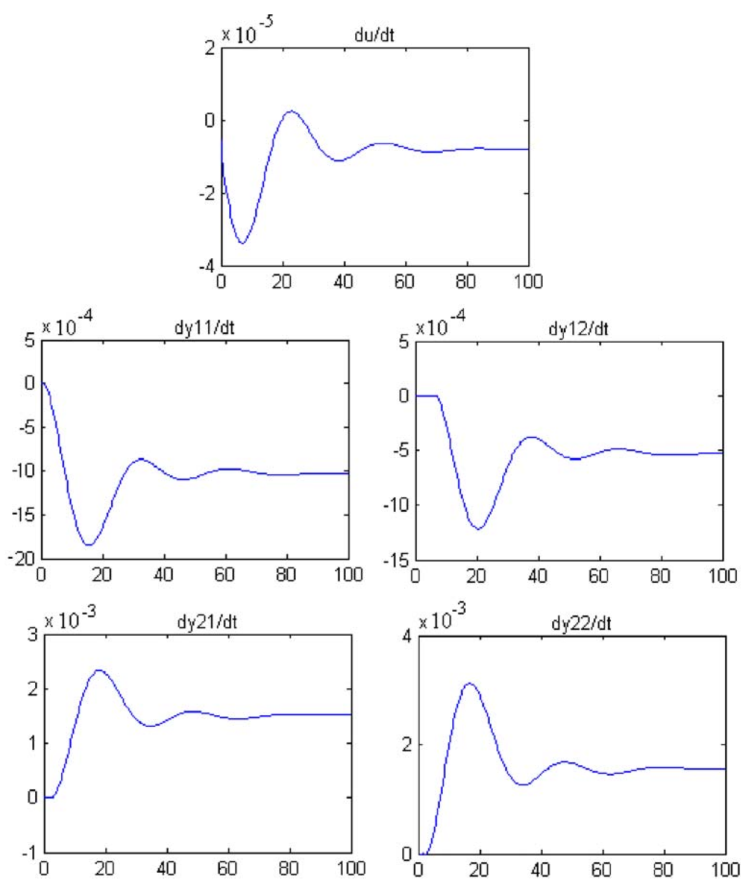

Fig. 4. Equivalent signals for the Wood & Berry model.

Table 1
Identification results under different noise level for example 1

Noise level (%)	$\hat{G}(s)$	$E(\%)$
0	$\hat{G}_{11}(s) = \frac{12.7929}{16.6193s+1}e^{-1.0044s}$	0.3573
	$\hat{G}_{12}(s) = \frac{-18.90}{21.001s+1}e^{-3.01s}$	0.0255
	$\hat{G}_{21}(s) = \frac{6.5998}{10.9205s+1}e^{-6.9852s}$	0.2749
	$\hat{G}_{22}(s) = \frac{-19.3942}{14.4339s+1}e^{-3.006s}$	0.6187
10	$\hat{G}_{11}(s) = \frac{12.7782}{16.6079s+1}e^{-1.1848s}$	4.2731
	$\hat{G}_{12}(s) = \frac{-18.7041}{20.6871s+1}e^{-2.8636s}$	2.4054
	$\hat{G}_{21}(s) = \frac{6.5809}{10.7352s+1}e^{-7.1670s}$	2.6477
	$\hat{G}_{22}(s) = \frac{-19.2724}{14.2769s+1}e^{-2.9083s}$	2.5753
20	$\hat{G}_{11}(s) = \frac{12.6057}{16.3221s+1}e^{-1.4046s}$	9.4847
	$\hat{G}_{12}(s) = \frac{-18.2795}{19.9567s+1}e^{-2.7695s}$	8.1978
	$\hat{G}_{21}(s) = \frac{6.5443}{10.4986s+1}e^{-7.3482s}$	6.6869
	$\hat{G}_{22}(s) = \frac{-19.0350}{13.9227s+1}e^{-2.8450s}$	7.6368

A complete process model has been developed in the literature (Bequette, 1998) under the assumptions that (i) only the component of interest is transferred from one phase to the other, (ii) the vapor stream leaving a stage is in thermodynamic equilibrium with the liquid on that stage, and (iii) the liquid holdup M in each tray is very large compared to the vapor holdup, which implies that the feed and product flow rates are at steady state. Furthermore, the thermodynamic equilibrium in the ith stage is represented by the expression $y_i = ax_i$, where a is an equilibrium parameter, and mass balances for stages $i = 2$ to $n-1$ yield the modeling equations

$$\frac{dx_i}{dt} = \frac{L}{M}x_{i-1} - \frac{L+Va}{M}x_i + \frac{Va}{M}x_{i+1}, \quad i = 1, 2, \ldots, n-1. \tag{27}$$

In turn, the feed tray ($i = 1$) and the product tray ($i = n$) are, respectively, modeled according to the mass balances

$$\frac{dx_1}{dt} = -\frac{L+Va}{M}x_1 + \frac{Va}{M}x_2 - \frac{L}{M}x_f \tag{28}$$

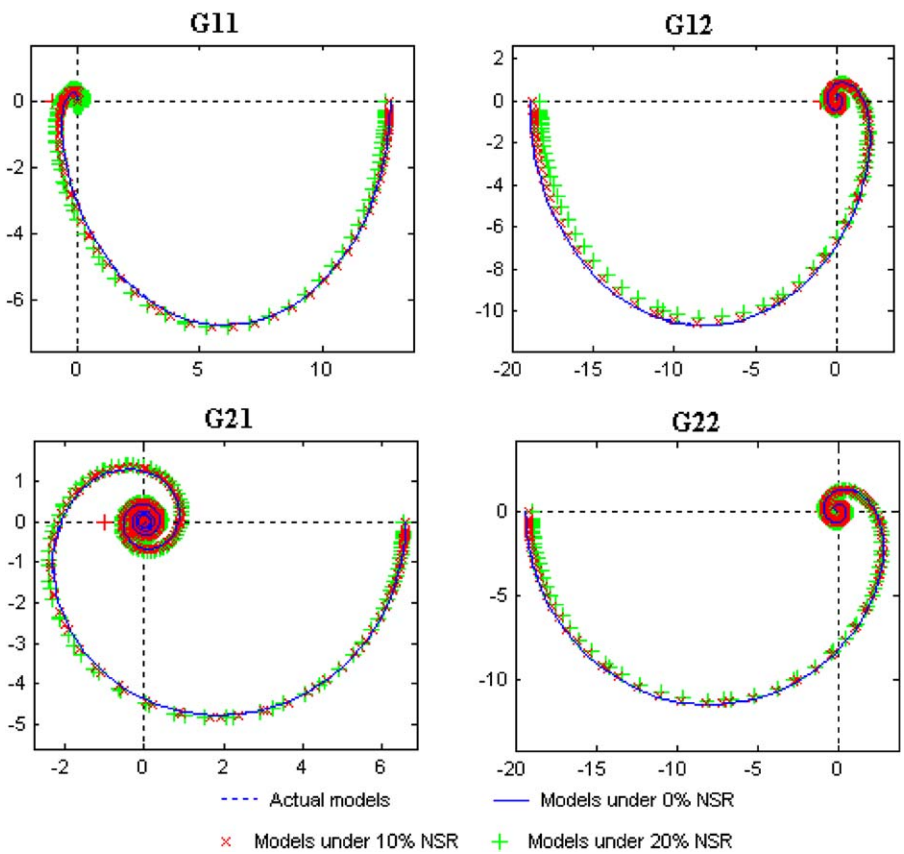

Fig. 5. *Nyquist* curves under different NSR.

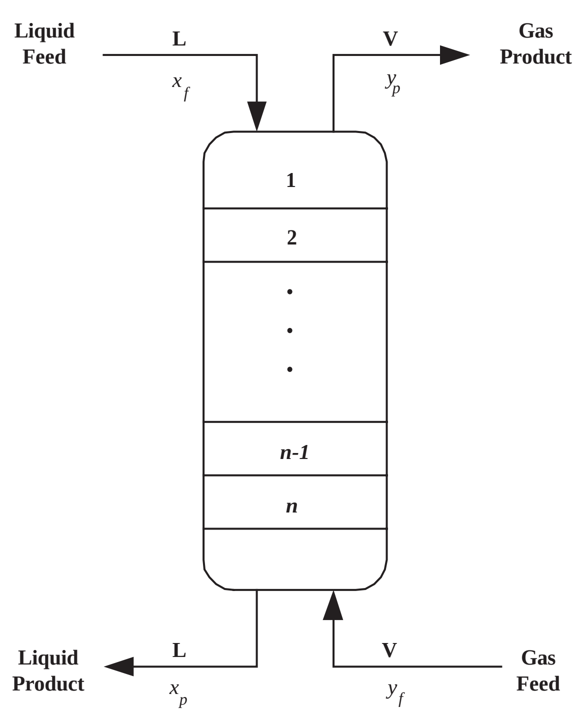

Fig. 6. Schematic of the gas absorption column with *n* equilibrium stages.

and

$$\frac{\mathrm{d}x_n}{\mathrm{d}t} = \frac{L}{M}x_{n-1} - \frac{L+Va}{M}x_n + \frac{V}{M}y_f. \tag{29}$$

Finally, the output equations are

$$x_p = x_n, \tag{30}$$

$$y_p = ax_1. \tag{31}$$

For the particular case of a five-tray column, model (27)–(31) reduces to the two-input/two-output, whose state-space representation is

$$\begin{bmatrix} \mathrm{d}x_1/\mathrm{d}t \\ \mathrm{d}x_2/\mathrm{d}t \\ \mathrm{d}x_3/\mathrm{d}t \\ \mathrm{d}x_4/\mathrm{d}t \end{bmatrix} = \begin{bmatrix} -0.325 & 0.125 & 0 & 0 & 0 \\ 0.200 & -0.325 & 0.125 & 0 & 0 \\ 0 & 0.200 & -0.325 & 0.125 & 0 \\ 0 & 0 & 0.200 & -0.325 & 0.125 \\ 0 & 0 & 0 & 0.200 & -0.325 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0.200 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.250 \end{bmatrix} \begin{bmatrix} x_f(t) \\ y_f(t) \end{bmatrix},$$

$$\begin{bmatrix} x_p(t) \\ y_p(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0.5 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix},$$

Table 2
Identification results under different noise level for example 1

Noise level (%)	$\hat{G}(s)$	ε
0	$\hat{G}_{11}(s) = \frac{1}{388.9s^2 + 68.42s + 2.508}e^{-5.13s}$	1.1291e-6
	$\hat{G}_{12}(s) = \frac{12.92s + 1}{143s^2 + 46.47s + 2.079}$	1.5949e-6
	$\hat{G}_{21}(s) = \frac{12.92s + 1}{57.19s^2 + 18.59s + 0.8318}$	6.0621e-6
	$\hat{G}_{22}(s) = \frac{1}{4028s^2 + 715.4s + 26.3}e^{-5.55s}$	3.9266e-8
20	$\hat{G}_{11}(s) = \frac{1}{451s^2 + 71.79s + 2.506}e^{-4.12s}$	9.1318e-6
	$\hat{G}_{12}(s) = \frac{13.92s + 1}{151.8s^2 + 48.77s + 2.079}$	4.599e-6
	$\hat{G}_{21}(s) = \frac{14.16s + 1}{60.87s^2 + 19.75s + 0.8314}$	2.7462e-5
	$\hat{G}_{22}(s) = \frac{1}{4148s^2 + 772.9s + 26.12}e^{-5.14s}$	5.8458e-7

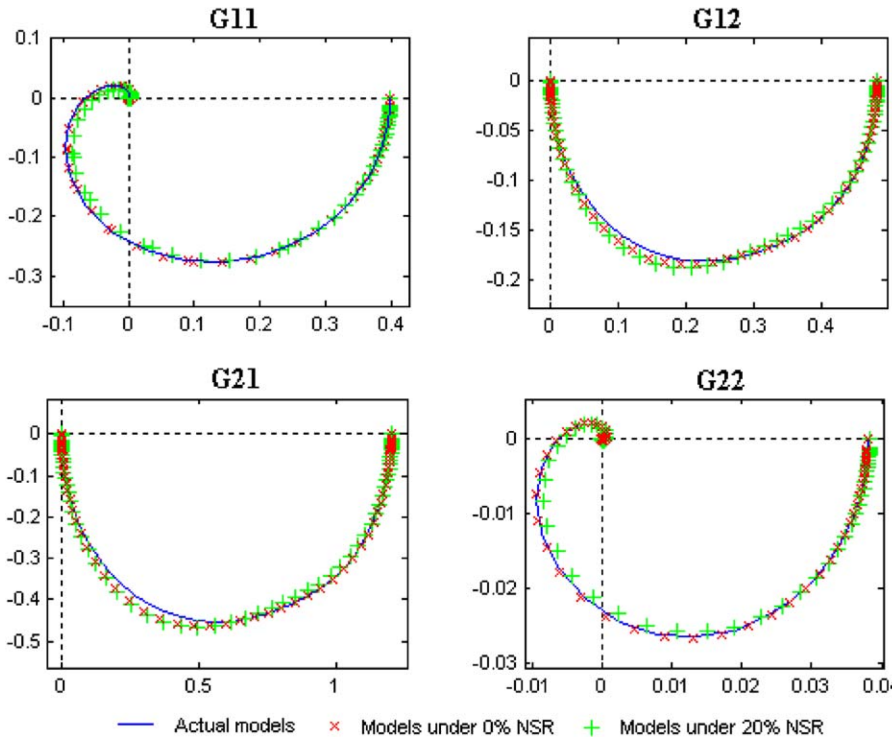

Fig. 7. *Nyquist* curves of 5 trays gas absorption column under different NSR.

where the system matrices are obtained after substituting in Eqs. (27)–(31). The equilibrium parameter $a = 0.5$ and the following steady-state values of process parameters: $L = 4/3$ kg mol/min, $V = 5/3$ kg mol/min, $M = 20/3$ kg mol/min, the steady-state input $x_{f,ss} = 0$ and $y_{f,ss} = 0.1$.

By selecting $K_1 = 1$ and $K_2 = 1$ for the closed-loop system, the obtained identification model under 0% NSR and 20% NSR respectively are given in Table 2, where a time domain identification error is defined as

$$\varepsilon = \frac{1}{N}\sum_{k=1}^{N}[y(kT_s) - \hat{y}(kT_s)]^2, \tag{32}$$

where $y(kT_s)$ is the actual process output under a step change, while $\hat{y}(kT_s)$ is the response of the estimated process under the same step change. Fig. 7 gives the *Nyquist* curves of those models under different NSR.

6. Conclusions

In this paper, a novel identification method based on step tests for multivariable process has been presented. By decoupling the closed-loop multivariable system into independent single open-loop systems with same input signal acting on the multiple transfer functions, most restrictions of existing multivariable process identification methods has been relaxed. The proposed method requires no prior knowledge of the process and the first-order or second-order plus delay model elements of the transfer function matrices can be directly obtained. The main advantages of the proposed methods are: (1) The testing procedure is straightforward, the computation is simple, and can be easily implemented; (2) It is valid for closed-loop tests; and (3) It can identify both close-coupled and weak-coupled processes. Various processes have been employed to demonstrate the effectiveness and practicability of the method. Simulation results show that the proposed identification method is practical and accurate even in the noisy environment, especially in the frequency range from zero to the frequency corresponding to π phase lag. It also showed that the time delays can be accurately approximated by first-order Taylor series, however, more accurate approximations are expected by frequency domain identification approach which is currently under investigation. Even though the method is derived for TITO systems, the extension to general MIMO system is straightforward.

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