Full-range behavior of FRP-to-concrete bonded joints

H. Yuan a,b, J.G. Teng a,*, R. Seracino c, Z.S. Wu d, J. Yao a

a Department of Civil and Structural Engineering, The Hong Kong Polytechnic University, Hong Kong, China
b Faculty of Construction, Guangdong University of Technology, Guangzhou, China
c School of Civil and Environmental Engineering, University of Adelaide, Adelaide, SA 5005, Australia
d Department of Civil and Urban Engineering, Ibaraki University, Hitachi 316-8511, Japan

Received 10 July 2003; received in revised form 24 November 2003; accepted 27 November 2003

Abstract

External bonding of fiber reinforced polymer (FRP) composites has become a popular technique for strengthening concrete structures all over the world. The performance of the interface between FRP and concrete is one of the key factors affecting the behavior of the strengthened structure, and has been widely studied using simple shear tests on FRP plate/sheet-to-concrete bonded joints. While a great deal of research is now available on the behavior of these bonded joints, no closed-form analytical solution has been presented which is capable of predicting the entire debonding propagation process. This paper presents such an analytical solution, in which the realistic bi-linear local bond–slip law is employed. Expressions for the interfacial shear stress distribution and load–displacement response are derived for different loading stages. It is also shown how experimental load–displacement responses of these joints can be used to quantify interfacial properties, including the interfacial fracture energy and parameters of the local bond–slip relationship. The debonding process is discussed in detail and the analytical results are compared with experimental data. Finally, results from the analytical solution are presented to illustrate how the bond length and the plate stiffness affect the behavior of such bonded joints. While the emphasis of the paper is on FRP-to-concrete joints, the analytical solution is equally applicable to similar joints between thin plates of other materials (e.g. steel and aluminum) and concrete.

#2003 Elsevier Ltd. All rights reserved.

Keywords: FRP, Concrete; Bond; Interface; Bond behavior; Bond strength; Effective bond length; Bond–slip model; Debonding; Interfacial fracture energy; Interfacial parameters; Analytical solution

1. Introduction

External bonding of fiber reinforced polymer (FRP) plates or sheets has emerged as a popular method for the strengthening or retrofitting of reinforced concrete (RC) structures [1–4]. In this strengthening method, the performance of the FRP-to-concrete interface in providing an effective stress transfer is of crucial importance. Indeed, a number of failure modes in FRP-strengthened RC members are directly caused by interfacial debonding between the FRP and the concrete. One of the failure modes, referred to as intermediate crack induced debonding (IC debonding), involves debonding which initiates at a major crack and propagates along the FRP-to-concrete interface. In RC beams flexurally-strengthened with a tension face FRP plate/sheet, IC debonding may arise at a major flexural crack or flexural-shear crack [5–7]. In RC beams shear-strengthened with FRP plates/sheets bonded to the sides, IC debonding can arise as a result of a shear crack [8]. In IC debonding, the interface is dominated by shear stresses, so the debonding failure is also referred to as Mode II fracture in the context of fracture mechanics. In RC beams bonded with a tension face plate, debonding is also likely at the plate ends where debonding is due to a combination of high shear stresses and high normal stresses [9–15]. It should be noted that while the emphasis of the paper is on FRP-to-concrete joints, the analytical solution is equally applicable to similar joints between thin plates of other materials (e.g. steel and aluminum) and concrete. Indeed, debonding failures of RC beams bonded with steel plates have also been studied extensively in the literature [16–18].
In IC debonding failures, the stress state of the interface is similar to that in a shear test specimen in which a plate is bonded to a concrete prism and is subject to tension (Fig. 1). As a result, a large number of studies, both experimental and theoretical, have been carried out on simple shear tests on bonded joints, with the earlier work being concerned with steel plates bonded to concrete. Experiments have been carried out using several set-ups, including single shear tests (e.g. [19–23]), double shear tests (e.g. [24–31]), and modified beam tests (e.g. [24,32,33]). Theoretical work has included fracture mechanics analysis [34–40], finite element analysis [41,42] and the development of design models [24,43–45].

Existing studies suggest that the main failure mode of FRP-to-concrete joints in shear tests is concrete failure under shear, occurring generally at a few millimeters from the concrete-to-adhesive interface. The ultimate load (i.e. the maximum transferable load) of the joint therefore depends strongly on concrete strength. In addition, the plate-to-concrete member width ratio also has a significant effect. A very important aspect of the behavior of these bonded joints is that there exists an effective bond length beyond which an extension of the bond length cannot increase the ultimate load. This is a fundamental difference between an externally bonded plate and an internal reinforcing bar for which a sufficiently long anchorage length can always be found that the full tensile strength of the reinforcement can be achieved. Existing studies have been mainly concerned with the prediction of the ultimate load and the effective bond length [45], but much less attention has been given to the prediction of the entire debonding process of such bonded joints. In particular, Chen and Teng [45] presented a review of most of the studies prior to theirs, assessed available models by comparison with experimental data gathered from the literature, and proposed accurate expressions for the ultimate load and the effective bond length. Chen and Teng’s model [45] was modified from an existing fracture mechanics model with suitable simplifications, and captures all the main features of anchorage behavior. It should be noted that the term ‘ultimate load’ is used in this paper instead of ‘bond strength’ to avoid confusion with the local bond strength of the interface.

Of the available closed-form analytical studies, the notable ones include Holzenkämpfer [35], Täljsten [46], Brosens and Van Gemert [37], Yuan et al.[39], and Wu et al. [40]. Holzenkämpfer [35] investigated the ultimate load of steel-to-concrete bonded joints using nonlinear fracture mechanics and demonstrated that sufficiently accurate results could be obtained using a bilinear bond–slip model. Täljsten [46] analytically obtained the expression for the ultimate load using a linear ascending bond–slip model. Brosens and van Gemert [37] derived expressions for the maximum transferable load at both the serviceability and the ultimate limit states. Yuan et al. [39] and Wu et al. [40] developed equations for the ultimate load, the interfacial shear stress distribution at the first attainment of this ultimate load and the effective bond length for various interfacial bond–slip models. However, no analytical solution has been presented which is capable of predicting the entire debonding propagation process.

This paper presents such an analytical solution, in which the realistic bi-linear local bond–slip model is employed, for the prediction of the entire debonding propagation process in FRP-to-concrete bonded joints. The authors believe that this analytical solution fulfills at least two important functions: (a) it provides a rigorous and complete theoretical basis for understanding the full-range load–displacement behavior of FRP-to-concrete bonded joints; and (b) it provides a method for identification of interfacial properties using experimental load–displacement responses. Both issues are important for the correct modeling of the FRP-to-concrete interface which is the key for the accurate prediction of the serviceability and ultimate behavior of FRP-strengthened RC members (e.g. [47,48]).

2. Governing equations

Fig. 1 shows a single-lap pull–push test of a plate-to-concrete bonded joint, in which the width and thickness of each of the three components (plate, adhesive layer and concrete prism) are constant along the length. The width and thickness of the plate are denoted by \( b_p \) and \( t_p \) respectively, those of the concrete prism by \( b_c \) and \( t_c \) respectively, and the bonded length of the plate (i.e. bond length) is denoted by \( L \). The
Young's modulus of the plate and concrete are $E_p$ and $E_c$ respectively. In such a joint, the adhesive layer is mainly subjected to shear deformations, so mode II interfacial fracture is the expected failure mode. A simple mechanical model for this joint can be thus established by treating the plate and the concrete prism (the two adherends) as being subject to axial deformations only while the adhesive layer can be assumed to be subject to shear deformations only. That is, both adherends are assumed to be subject to uniformly distributed axial stresses only, with any bending effects neglected, while the adhesive layer is assumed to be subject to shear stresses only which are also constant across the thickness of the adhesive layer. It should be noted that in such a model, the adhesive layer represents not only the deformation of the actual adhesive layer but also that of the materials adjacent to the adhesive layer and is thus also referred to in the paper as the interface. Based on these assumptions, the following fundamental equations can be easily found [40] based on equilibrium considerations (Fig. 2):

\[
\frac{d \sigma_p}{dx} - \frac{\tau}{t_p} = 0 \quad (1)
\]
\[
\sigma_p t_p b_p + \sigma_c t_c b_c = 0 \quad (2)
\]

where $\tau$ is the shear stress in the adhesive layer, $\sigma_p$ is the axial stress in the plate and $\sigma_c$ is the axial stress in the concrete prism.

The constitutive equations for the adhesive layer and the two adherends can be expressed as

\[
\tau = f(\delta) \quad (3)
\]
\[
\sigma_p = E_p \frac{d \delta_p}{dx} \quad (4)
\]
\[
\sigma_c = E_c \frac{d \delta_c}{dx} \quad (5)
\]

The interfacial slip $\delta$ is defined as the relative displacement between the two adherends, that is

\[
\delta = u_p - u_c \quad (6)
\]

After substituting Eqs. (2)-(6) into Eq. (1) and introducing the parameters of local bond strength $\tau_f$ and interfacial fracture energy $G_f$ yield the following

\[
\frac{d^2 \delta}{dx^2} - \frac{2G_f}{\tau_f^2} \lambda^2 f(\delta) = 0 \quad (7)
\]

and

\[
\sigma_p = -\frac{\tau_f^2}{2G_f} \frac{d \delta}{dx} \quad (8)
\]

where

\[
\lambda^2 = \frac{\tau_f^2}{2G_f} \left( \frac{1}{E_p t_p} + \frac{b_p}{b_c E_c} \right) \quad (9)
\]

Eq. (7) is the governing differential equation of the bonded joint shown in Fig. 1 and can be solved if the local bond–slip model relating the local interfacial shear stress to the local shear slip as represented by $f(\delta)$ is defined. The interfacial fracture energy, which is simply the area under the local bond–slip curve, is introduced because once it is known it can be used regardless of the exact shape of the local bond–slip curve where a particular quantity (e.g. the ultimate load) depends on the interfacial fracture energy but not on the shape of the bond–slip curve.

3. Local bond–slip model

Various bond–slip models have been considered in previous work [3]. However, experimental results (e.g. [48,49]) indicate that the bilinear model shown in Fig. 3 which features a linear ascending branch followed by a linear descending branch provides a close approximation. According to this model, the bond shear stress increases linearly with the interfacial slip until it reaches the peak stress $\tau_f$ at which the value of the slip

Fig. 2. Deformation and equilibrium in a bonded joint.

Fig. 3. Local bond–slip model.
is denoted by $\delta_1$. Interfacial softening (or micro-cracking) then starts with the shear stress reducing linearly with the interfacial slip. The shear stress reduces to zero when the slip exceeds $\delta_f$, signifying the shear fracture (or debonding or macro-cracking) of a local bond element. The absence of any residual shear strength after debonding implies that friction and aggregate interlock over the debonded length of the joint is ignored. This bond–slip model shown in Fig. 3 is mathematically described by the following

$$f(\delta) = \begin{cases} \frac{\tau_f}{\delta_1} \delta & \text{when } 0 \leq \delta \leq \delta_1 \\ \frac{\delta_1 - \delta}{\delta_f} (\delta_f - \delta) & \text{when } \delta_1 < \delta \leq \delta_f \\ 0 & \text{when } \delta > \delta_f \end{cases}$$  \hspace{1cm} (10)

4. Analysis of the debonding process

With the bond–slip model defined above, the governing equation (Eq. (7)) can be solved to find the shear stress distribution along the interface and the load–displacement response of the bonded joint. The solution is presented below stage by stage with illustrations of the corresponding interfacial shear stress distribution (Fig. 4) and load–displacement (Fig. 5). The interfacial shear stress distribution shown in Fig. 4 is strictly correct only for an infinite bond length, although the theoretical predictions are indistinguishable for a bond length substantially higher than the effective bond length for the transfer of the ultimate load (Eq. (42)) (say twice this effective bond length or higher). The load–displacement curve shown in Fig. 5 is for a joint with a bond length substantially higher than this effective bond length, with a longer plateau corresponding to a larger bond length. The above conditions should be kept in mind when referring to Figs. 4 and 5 but will not be further emphasized. The solution presented here is however valid for all bond lengths.
4.1. Elastic stage

At small loads, there is no interfacial softening or debonding along the plate-to-concrete interface, so the entire length of the interface is in an elastic stress state (state I) (Fig. 4a). This is true as long as the interfacial shear stress at $x = L$ is less than $\tau_f$. Substituting the relationship of Eq. (10) for the case of $\delta \leq \delta_1$ into Eq. (7), the following differential equation is obtained

$$\frac{d^2 \delta}{dx^2} - \lambda_1^2 \delta = 0$$

(11)

where

$$\lambda_1^2 = \frac{2G_{lf}}{\tau_f \delta_1} = \frac{1}{E_{lp} + \frac{b_p}{b_c E_{lc}}}$$

(12)

and the boundary conditions are

$$\sigma_p = 0 \text{ at } x = 0$$

(13)

$$\sigma_p = \frac{P}{b_p} \text{ at } x = L$$

(14)

With these boundary conditions the following expressions for the interfacial slip, the interfacial shear stress and the axial stress in the plate are found by solving Eq. (11)

$$\delta = \delta_1 \frac{P_{12} \cosh(\lambda_1 x)}{\tau_f b_p \sinh(\lambda_1 L)}$$

(15)

$$\tau = \frac{P_{12} \cosh(\lambda_1 x)}{b_p \sinh(\lambda_1 L)}$$

(16)

$$\sigma_p = \frac{P \sinh(\lambda_1 x)}{b_p \sinh(\lambda_1 L)}$$

(17)

During this stage of loading, the interfacial shear stress distribution is of the form shown in Fig. 4a with the maximum bond shear stress being smaller than the local bond strength $\tau_f$. The slip at the loaded end (i.e. the value of $\delta$ at $x=L$) is defined as the displacement of the bonded joint and is denoted by $\Delta$. Making use of this definition, the following load–displacement relationship can be obtained from Eq. (15):

$$P = \frac{\tau_f b_p}{\lambda_1} \frac{\Delta}{\delta_1} \tanh(\lambda_1 L)$$

(18)

Introducing the following two non-dimensional parameters

$$\mathcal{P} = \frac{P}{\tau_f b_p} \text{ and } \tilde{\alpha} = \frac{\Delta}{\delta_1}$$

(19)

Eq. (18) can be simplified to

$$\mathcal{P} = \tilde{\alpha} \frac{\delta_1}{\delta_1} \tanh(\lambda_1 L) \text{ for } 0 \leq \tilde{\alpha} \leq \frac{\delta_1}{\delta_1}$$

(20)

The typical full-range theoretical load–displacement curve for a bonded joint as shown in Fig. 1 is given in

Fig. 5. Eq. (20) depicts the linear load–displacement relationship in the elastic stage of loading, which is shown as segment OA in Fig. 5.

It should be noted that during linear elastic deformation, only part of the interface is significantly stressed with the stresses elsewhere being very small (Fig. 4a). The length of the interface that is mobilized to resist the applied load is generally referred to as the effective bond length. Based on this definition and considering that $\tanh(2) \approx 0.97$, the effective bond length during elastic deformation is independent of the load level and is given by

$$l_{e,e} = \frac{2}{\lambda_1}$$

(21)

The elastic stage of deformation of the bonded joint ends when the shear stress reaches the local shear strength $\tau_f$ at a slip of $\delta_1$ at $x = L$ (Fig. 4b). Setting $\Delta = \delta_1$ in Eq. (18) leads to the load at the initiation of interfacial softening given by

$$P_s = \frac{\tau_f b_p}{\lambda_1} \tanh(\lambda_1 L)$$

For an infinite bond length, Eq. (22) converges to

$$P_s = \frac{\tau_f b_p}{\lambda_1}$$

(23)

4.2. Elastic-softening stage

Once the shear stress reaches $\tau_f$ at $x = L (\Delta = \delta_1)$, softening commences at the loaded end of the bonded plate, so part of the plate-to-concrete interface enters the softening state (state II) while the rest remains in the elastic state (state I) as shown in Fig. 4c. The load $P$ continues to increase as the length of the softening zone $a$ increases. The ultimate load $P_u$ is first attained at the end of this stage. Substituting the relationship given in Eq. (10) into Eq. (7) gives the following for the elastic-softening stage

$$\frac{d^2 \delta}{dx^2} = -\lambda_2^2 \delta$$

for $0 \leq \delta \leq \delta_1$

(24)

$$\frac{d^2 \delta}{dx^2} + \lambda_2^2 \delta = \lambda_2^2 \delta_1$$

for $\delta_1 < \delta \leq \delta_t$

(25)

where

$$\lambda_2^2 = \frac{2G_{lf}}{(\delta_1 - \delta_1)\tau_f} = \frac{\tau_f}{\delta_1 - \delta_1} \left( \frac{1}{E_{lp} + \frac{b_p}{b_c E_{lc}}} \right)$$

(26)

The solutions to Eqs. (24) and (25) are of similar form to Eqs. (15)–(17) and can be derived using
the following boundary conditions

\[ \sigma_p = 0 \quad \text{at} \quad x = 0 \]  (27)

\[ \sigma_p \text{ is continuous} \quad \text{at} \quad x = L - a \]  (28)

\[ \delta = \delta_1 \quad \text{or} \quad \tau = \tau_f \quad \text{at} \quad x = L - a \]  (29)

\[ \sigma_p = \frac{P}{l_p b_p} \quad \text{at} \quad x = L \]  (30)

The solution for the elastic region of the interface \([0 \leq \delta \leq \delta_1, \text{i.e.} \ 0 \leq x \leq L - a]\) is given by

\[ \delta = \delta_1 \frac{\cosh(\lambda_1 x)}{\cosh(\lambda_1(L - a))} \]  (31)

\[ \tau = \tau_f \frac{\cosh(\lambda_1 x)}{\cosh(\lambda_1(L - a))} \]  (32)

\[ \sigma_p = \frac{\tau_f}{\lambda_1}, \quad \text{tanh}(\lambda_1 x) \]  (33)

and the solution for the softening region of the interface \([\delta_1 < \delta \leq \delta_1, \text{i.e.} \ L - a \leq x \leq L]\) is given by

\[ \delta = (\delta_t - \delta_1) \left\{ \frac{\lambda_2}{\lambda_1} \tanh[\lambda_1(L - a)] \right\} \]  (34)

\[ \times \sin[\lambda_2(x - L + a)] - \cos[\lambda_2(x - L + a)] + \frac{\delta_1}{\delta_t - \delta_1} \right\} \]

\[ \tau = -\tau_f \left( \frac{\lambda_2}{\lambda_1} \tan[\lambda_1(L - a)] \right) \sin[\lambda_2(x - L + a)] - \cos[\lambda_2(x - L + a)] \]  (35)

\[ \sigma_p = \frac{\tau_f}{\lambda_2} \left\{ \frac{\lambda_2}{\lambda_1} \tanh[\lambda_1(L - a)] \right\} \cos[\lambda_2(x - L + a)] + \sin[\lambda_2(x - L + a)] \]  (36)

Substituting Eq. (30) into Eq. (36) yields

\[ \bar{\tau} = \frac{\lambda_1}{\lambda_2} \frac{\lambda_2}{\lambda_1} \tanh[\lambda_1(L - a)] \cos(\lambda_2a) + \sin(\lambda_2a) \]  (37)

and the displacement at \(x = L\) can be obtained from Eq. (34) such that

\[ \bar{\delta} = \left( \delta_t - \delta_1 \right) \left\{ \frac{\lambda_2}{\lambda_1} \tan[\lambda_1(L - a)] \right\} \sin(\lambda_2a) - \cos(\lambda_2a) + \frac{\delta_1}{\delta_t - \delta_1} \right\} \]  (38)

The distribution of the interfacial shear stress during the elastic-softening stage is illustrated in Fig. 4c. During this stage, the load-displacement curve plotted from Eqs. (37) and (38) is shown as segment AB in Fig. 5. Obviously, the joint softens during this stage and reaches its ultimate load at the end of this stage. \(P\) reaches its maximum when the derivative of Eq. (37) with respect to \(a\) equals zero. Therefore, \(a\) at the ultimate load can be found from the following relationship

\[ \tanh[\lambda_1(L - a)] = \frac{\lambda_2}{\lambda_1} \tan(\lambda_2a) \]  (39)

Substituting Eq. (39) into Eq. (37) yields

\[ P = \frac{\tau_f b_p}{\lambda_2}, \quad \frac{\lambda_1}{\delta_t - \delta_1} \sin(\lambda_2a) \]  (40)

In general, \(a\) can only be found from Eq. (39) by iteration. However, it can be shown that for infinite bond lengths, \(P\) reaches its maximum when \(\tau = 0\) at \(x = L\) and Eq. (40) converges to

\[ P = \frac{\tau_f b_p}{\lambda_2} \]  (41)

Defined in the same manner as in Eq. (21), the effective bond length when \(P_u\) is reached can be obtained from Eqs. (39) and (40) to give

\[ l_e = a + \frac{1}{2\lambda_1} \ln \frac{1 + \lambda_2 \tan(\lambda_2a)}{1 - \lambda_2 \tan(\lambda_2a)} \]  (42)

where

\[ a = \frac{1}{\lambda_2} \arcsin \left[ 0.97 \left( \frac{\delta_t - \delta_1}{\delta_t} \right) \right] \]  (43)

4.3. Elastic-softening-debonding stage

During this stage of loading, debonding (or macro-cracking or fracture) commences and propagates along the interface. At the initiation of debonding \(\Delta = \delta_t\) and by making use of this condition, the corresponding value of \(a\), denoted by \(a_d\), can be obtained from Eq. (38) as

\[ \frac{\lambda_2}{\lambda_1} \tan[\lambda_1(L - a)] \right\} \sin(\lambda_2a) - \cos(\lambda_2a) = 0 \]  (44)

For infinite bond lengths, Eq. (44) reduces to

\[ a_d = \frac{1}{\lambda_2} \arctan \left( \frac{\lambda_1}{\lambda_2} \right) \]  (45)

The interfacial shear stress distribution along the interface at the initiation of debonding is shown in Fig. 4d. As debonding propagates, the peak shear stress \(\tau_f\) moves towards \(x = 0\), the unloaded end of the bonded plate. Depending on the location, the plate-to-concrete interface during debonding propagation is in one of three possible stress states: the elastic state (state I), the softening state (state II) and the stress-free debonded state (state III) (Fig. 4c). Assuming that the debonded length of the interface starting at the loaded end of the plate is \(d\), Eqs. (31) and (36) are still valid if \(L\) is replaced by \((L - d)\). Therefore, the load-
The displacement relationship can be written as

\[
P = \frac{\lambda}{\lambda_2} \left\{ \frac{\lambda_2}{\lambda_1} \tanh[\lambda_1(L - d - a)] \cos(\lambda_2 a) + \sin(\lambda_2 a) \right\}
\]  

(46)

\[
\bar{x} = 1 + P\lambda d
\]

(47)

As the interfacial shear stress at \(x = L - d\) is zero, the following equation relating \(a\) to \(d\) can be obtained

\[
\frac{\lambda_2}{\lambda_1} \tanh[\lambda_1(L - d - a)] \sin(\lambda_2 a) - \cos(\lambda_2 a) = 0
\]

(48)

Substituting Eq. (48) into Eq. (46) yields the following simplified form

\[
P = \frac{\lambda}{\lambda_2 \sin(\lambda_2 a)}
\]

(49)

This stage is represented by segment BCD of the load–displacement curve shown in Fig. 5. Point C corresponds to the deformation state at which the transferable load starts to reduce because the interfacial stress distribution is now truncated by the free end. At the end of this stage at point D, the softening-debonding stage begins when \(L - d = a_u\). Substituting this relation into Eq. (48) yields

\[
a_u = \frac{\pi}{2\lambda_2}
\]

(50)

Moreover, Eq. (49) can be rewritten as

\[
P = \frac{\tau_1 b_0}{\lambda_2}
\]

(51)

and the interface shear stress distribution at the end of this stage is shown in Fig. 4f.

4.4. Softening-debonding stage

The softening-debonding stage is governed by Eq. (25) with the following boundary conditions:

\[
\sigma_p = 0 \quad \text{at} \quad x = 0
\]

(52)

\[
\delta = \delta_1 \quad \text{and} \quad \sigma_p = \frac{P}{t_b h_p} \quad \text{at} \quad x = a
\]

(53)

The following solution can thus be found

\[
a = a_u = \frac{\pi}{2\lambda_2}
\]

(54)

\[
\delta = \delta_1 - \frac{P \delta_1 \lambda_2^2}{b_1 \tau_1 \lambda_2} \cos(\lambda_2 x) \quad \text{at} \quad 0 \leq x \leq a_u
\]

(55)

Eq. (54) shows that the softening zone length remains constant during the softening-debonding stage. During this stage, the maximum interfacial shear stress at \(x = 0\) reduces with the load (Fig. 4g). The displacement at the loaded end can be obtained by solving Eq. (7) for the case of \(\delta > \delta_1\), or directly and more simply by displacement superposition along the bonded joint giving the following load–displacement relationship

\[
\bar{\Delta} = 1 + P\lambda(L - a_u)
\]

(56)

Eq. (56) indicates that the displacement reduces linearly with the load as shown by segment DE of the load–displacement curve in Fig. 5.

4.5. Characteristic points of the load–displacement curve

As discussed above, the entire load–displacement curve (Fig. 5) consists of a number of distinct segments corresponding to distinct stages of loading. Of particular importance are point A at the initiation of interfacial softening and point B at the initiation of debonding (first attainment of the ultimate load). These two points can be obtained without much difficulty from an experimental load–displacement curve and can be used to identify the necessary parameters of the interface. Point C at the end of the plateau, point D when the peak shear stress reaches the unloaded end of the bonded plate and point E at complete debonding failure of the joint are generally not obtainable from experiment. As a result, the present analysis provides a useful tool for the determination of the parameters of the bilinear bond–slip model from simple tests on bonded joints as illustrated in the following section.

5. Comparison of analytical solution with experimental results

5.1. Load–displacement curve

A series of pull-push shear tests on single-lap bonded joints were recently carried out by Yao [50] to determine the effect of various parameters on the bond behaviour of FRP plate-to-concrete joints. The thin FRP plates were instrumented with strain gauges at 10 mm intervals along the bond length \(L\) to monitor the variation of plate strains with load. The results showed that the plate strains in the debonded zone were substantially affected by plate bending due to the thinness of the plate and the roughness of the cracked interface. Since the bending components of plate strains could not be isolated in these tests as strain gauges were installed on only one surface of the plate, for comparison with the analytical solution presented in this paper, specimen II-5 was selected because the plate strain distribution within the debonded zone in this specimen was least affected by plate bending. It should be noted that while the bending strains in the plate can become significant, the interfacial debonding behaviour including debonding propagation and load–displacement response is believed to be dominated by the membrane component of plate strains, as implied by the assumptions of the present analytical solution.
The experimental set-up simulated the loading condition depicted by Fig. 1 and the material and geometric properties are as follows: \( t_p = 0.165 \text{ mm (nominal thickness)}, \ b_p = 25 \text{ mm}, \ t_c = 150 \text{ mm}, \ b_c = 150 \text{ mm}, \ L = 190 \text{ mm}, \ E_p = 256,000 \text{ MPa} \) and \( E_c = 28,600 \text{ MPa} \). In addition, the concrete cube compressive strength = 29 MPa. The use of the nominal plate thickness can be justified by noting that in the present solution, provided the axial stiffness of the plate per unit width \( (E_p t_p) \) is the same, a different definition of the plate thickness has no consequence. Indeed, this is a desirable feature of the solution, as otherwise, the definition of plate thickness for wet lay-up plates complicates the problem.

Experimental plate strain distributions of specimen II-5 for various load levels are shown in Fig. 6. The total displacement of the plate relative to the concrete at the loaded end (i.e. \( \Delta \)) was calculated by integrating the strain distributions using Simpson’s rule where possible supplemented by the trapezoidal rule. In obtaining the experimental load-displacement response, it was assumed that the slip between the plate and the concrete and the plate strain are zero at \( x = 0 \), which is reasonable except that at the later stages of loading the first assumption is not strictly valid when the debonding crack approaches the unloaded end. The experimental load-displacement response so obtained for specimen II-5 is shown as the dark dots in Fig. 7.

To determine the interfacial parameters of the bond-slip model for this specimen, points A \( (u_1, P_1) \) and B \( (u_2, P_2) \) as defined in Fig. 5 are identified on the experimental load-displacement curve as shown in Fig. 7. Based on the analytical solution presented earlier in the paper and by assuming that the axial stiffness of the plate is much smaller than that of the concrete (i.e. \( (EA)_p/(EA)_c \approx 0 \)), the following expressions can be obtained

\[
\delta_f = u_2 \tag{57}
\]

\[
\tau_f = \frac{P^2}{E_p t_p b_p^2 \tau_f} \tag{58}
\]

\[
\delta_1 = u_1 \tag{59}
\]

and the following redundant equation can be used to verify the accuracy

\[
\delta_1 = \frac{P^2}{E_p t_p b_p^2 \tau_f} \tag{60}
\]

The interfacial fracture energy can be obtained using the following, which is the area under the bond-slip curve

\[
G_f = \frac{1}{2} \tau_f \delta_f \tag{61}
\]

According to the experimental load-displacement curve, the interfacial parameters were identified as: \( \delta_f = 0.16 \text{ mm}, \ \delta_1 = 0.034 \text{ mm}, \ \tau_f = 7.2 \text{ MPa}, \) and \( G_f = 0.58 \text{ N/mm} \). From Eq. (60), \( \delta_1 = 0.027 \text{ mm} \), which is compared with 0.034 mm from Eq. (59). Although the two values for \( \delta_1 \) differ by approximately 25%, the effect on the predicted behavior is negligible. Using these values, the theoretical load-displacement curve was obtained and is shown as the solid line in Fig. 7. It is clear that the present analysis based on the above interfacial parameters provides a close prediction of the experimental load-displacement behaviour. The increase in load observed experimentally after the initiation of debonding due to aggregate interlock and friction in the debonded zone is not considered in the present analysis. The theoretical value of \( \Delta \) at failure was calculated to be 0.74 mm.
5.2. Interfacial shear stress distribution

A comparison of the interfacial shear stress distributions at various stages of loading is shown in Fig. 8, where the thick solid lines represent the theoretical distributions and the dots connected with a thin solid or dashed line represent the average experimental shear stress between two strain gauges calculated using the following relationship

\[
\tau_{\text{avg}} = \frac{E_p t_p}{s} \left[ (\epsilon_p)_{i+1} - (\epsilon_p)_{i-1} \right]
\]

where \(s\) is the spacing between strain gauges \((i+1)\) and \((i-1)\).

Fig. 8a shows the distribution at an applied load of 2.26 kN corresponding to point A in Fig. 7 immediately prior to softening such that the joint is entirely in an elastic state. The agreement is very good as might be expected, except for the strain reading near the loaded end. Fig. 8b shows the distributions for a load of 5.53 kN, corresponding to point B in Fig. 7 at the initiation of debonding such that the end of the theoretical softening branch is at the loaded end. The agreement between the theoretical and the experimental distributions is less satisfactory, due to the large fluctuations in the experimental shear stresses, but the theoretical distribution is seen to provide a trend line for the experimental results away from the loaded end. Identified by crosses in Fig. 8b is the experimental distribution for a load of 4.75 kN, where it can be seen that local debonding had occurred even at this lower load. The comparisons shown in Fig. 8b indicate that due to local stress concentration near the loaded end which is not considered in the present analytical solution, localized debonding occurred near the loaded end at a low load and this led to a rather different shear stress distribution near the loaded end. Finally, Fig. 8c shows the distributions when debonding propagated by 55 mm from the loaded end, corresponding to an experimental load of 5.74 kN, identified as point F in Fig. 7. The extent of debonding can be determined either from the experimental interfacial shear stress distribution where the stress along the softening branch of the distribution approaches zero (Fig. 8c), or from the plate axial strain distribution where the strain begins to plateau (Fig. 6). The theoretical distribution obtained using \(d = 55\) mm is seen to be in good agreement with the experimental results away from the debonded zone. The theoretical shear stress is zero where debonding has occurred but the experimental shear stress shows large fluctuations around the zero value. This can be explained by examining the plate axial strain distribution corresponding to \(P = 5.74\) kN in Fig. 6, which was used to determine the experimental shear stresses in Fig. 8c. The plate strains along the plateau, representing the extent of debonding, fluctuate as a result of plate bending due to the thinness of the plate and the roughness along the crack. If thicker steel or pultruded FRP plates with an increased resistance to bending are used, a more uniform shear stress distribution can be expected. The theoretical displacement \(\Delta\) calculated for this load is 0.45 mm which is in good agreement with the experimental displacement of 0.34 mm (point F in Fig. 7).
5.3. Plate axial stress distribution

Fig. 9 shows similar comparisons for plate axial stress distributions. Compared to the comparisons for shear stresses (Fig. 8), the theoretical predictions for axial stresses are in closer agreement with the experimental results in terms of the overall trend, despite the large fluctuations in the experimental results. In the debonded zone, the theoretical axial stress is constant, while the experimental axial stress fluctuates around the theoretical value.

6. Parametric study

The effects of bond length and FRP plate stiffness per unit width \( (E_{ptp}) \) on the load–displacement response, ultimate load and effective bond length are illustrated in Figs. 10 and 11 using the experimental joint as the reference joint. That is, except for the parameter being varied, the values of all other parameters are the same as those given in the preceding section.

Fig. 10a shows that the bond length has a great influence on the load–displacement curve. A longer bond length improves the ductility of the failure process; that is, the displacement of the plate at the loaded end increases while the load is maintained at the ultimate load. Fig. 10b shows a plot of ultimate load against total bond length. It is clear that the ultimate load increases with the bond length until an effective bond length \( l_e \) is reached, beyond which the ultimate load remains unchanged.

![Plate axial stress distributions in a test specimen.](image1)

Fig. 9. Plate axial stress distributions in a test specimen. (a) Initiation of softening, \( P = 2.26 \text{ kN} \); (b) Initiation of debonding, \( P = 5.53 \text{ kN} \); (c) Length of debonded interface = 55 mm.

![Effect of bond length.](image2)

Fig. 10. Effect of bond length. (a) Load-displacement behavior; (b) Ultimate load.
Fig. 11a shows that increasing the stiffness of the plate increases the ultimate load with a significant reduction in ductility. Fig. 11b illustrates more directly the effect of plate stiffness on ultimate load. Finally, Fig. 11c demonstrates that a stiffer plate leads to a longer effective bond length.

7. Conclusions

In this paper, a closed-form analytical solution has been presented to predict the entire debonding process of FRP-to-concrete bonded joints. In this solution, the realistic bi-linear local bond–slip model is employed. The solution provides closed-form expressions for the interfacial shear stress distribution and load–displacement response for different loading stages. As a result, the solution provides a rigorous and complete theoretical basis for understanding the full-range load–displacement behavior of FRP-to-concrete bonded joints. It should be noted that while the emphasis of the paper is on FRP-to-concrete joints, the analytical solution is equally applicable to similar joints between thin plates of other materials (e.g. steel and aluminum) and concrete. Based on the results and discussions presented in the paper, the following conclusions may be drawn:

(a) The load–displacement behavior of a bonded joint features a linear elastic stage, a softening stage, a debonding propagation stage, and a linear unloading stage;
(b) The ductility of the load–displacement behavior of a bonded joint increases with the bond length but decreases with the plate axial stiffness;
(c) The ultimate load of bonded joints increases with the bond length before the effective bond length is reached and remains constant afterwards. It also increases with the plate axial stiffness.
(d) The analytical solution provides a method for identification of interfacial properties for a bilinear local bond–slip model using data of two key points on the experimental load–displacement response. The predictions based on such an identified bond–slip model have been shown to be in close overall agreement with experimental results.

Acknowledgements

The authors gratefully acknowledge the financial support provided by The Hong Kong Polytechnic University through the Area of Strategic Development (ASD) Scheme and the Natural Science Foundation of China (National Key project no. 50238030).

References


