# 螺旋轨道的相对论天体动力学、局部性等效原理及银河系的暗物质问题

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**摘要：**本文关于相对论天体动力学问题的研究结果表明，由于引力传播速度的有限性导致的相对论光行差效应，使得银河系外缘天体围绕银心的运动轨迹为对数螺线，因此形成了平直的星系自转曲线和表观的宇宙暗物质现象。本文推论出物体的惯性质量与引力质量的等值性关联在大尺度上被切断，因此修正牛顿动力学的经验定律（MOND）得以在理论上成立，先驱者号宇宙飞船的轨道异常则可以得到明确的解释。应用这些理论，本文推导出星系引力透镜效应的新公式，该公式表明，在星系可见质量的外缘，光线在星系引力场中的偏折角度是一个常数，与碰撞参数无关。恒定的光线偏折角度曲线可以延伸至一个孤立星系的最边缘处，其规律恰好与平直的星系速度旋转曲线相符合，二者都可以造成一种假象，似乎有某种分布均匀但看不见的暗物质晕环绕在星系外围。

**关键词：**暗物质，天体动力学，等效原理，星系旋转曲线，对数螺旋轨道，引力透镜

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# The relativistic astrodynamics of spiral tracks, localized equivalence principle and the dark matter problem of our Milky Way galaxy

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**Abstract:** The relativistic astrodynamics of spiral track is discussed in this paper. This study shows that, in outer regions of the Milky Way galaxy, the relativistic effects, in particular, the aberration effect of relativity are the causes of logarithmic spiral tracks of the celestial bodies, which result in the flat rotation curves of the galaxy and the phenomena of dark matter in universe. It is derived that, at large galactic and cosmological distances, the equivalence relation of the inertia mass to the gravitational mass of an object has been cut off. These results present a theoretical proof for MOND and a clear explanation to the Pioneer anomaly. A new formula of the galaxy's gravitational lensing is derived by applying these

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theories, it shows a flat curve of deflection angles in the outer parts of the galaxy instead of a downward curve with increase of the impact parameter. The flat curve of constant deflection angle of an isolated galaxy lens would stretch away as far as the border of the galaxy, which agrees fairly well with the flat rotation curves of stars in the galaxy. Both approaches would show the same illusion of some sort of uniform but unseen dark matter halo around the galaxy.
**Key words:** Dark matter, Astrodynamics, Equivalence principle, Rotation curves, Logarithmic spiral track, Gravitational lensing.

# 0 Introduction

Dark matter problem is a big mystery in astronomy. In the observable regions of the outskirt of galaxies, the rotation curves of stars appear to be straight lines[1, 2, 3, 4]. This is a universal phenomenon deviated from Newton's theory and no exception in galaxies, without the extra matters or the extra gravity, there would be no sense of it. However, in the local system such as the solar system, the rotation curves of all planetary motion have no such behaviors, which are consistent with Newton's theory.

There are only two influential hypotheses trying to acquire the extra matters or the extra gravity, which are totally distinct kinds of candidates on the matter in academia. The first and the most popular explanation is the thought that there exists currently unobservable matter in universe, these unobservable matter would supplement the missing mass in the cosmic virial theorem, which ensures the perfection of Newton's principle. Although in order to explain the hypothesis of dark matter, astronomers have done numerous astronomical observations, up to now, no one have actually found any dark matter. The other searches for dark matter particles are disappointing as well: no such particles have yet been directly detected.[5, 6]

Another kind of explanations is that the so called unobservable dark matter in the galaxy does not exist, and it is Newtonian dynamics itself untenable at the edge of galaxy. There are numerous studies to discuss the possibilities of the alternatives to dark matter. The most influential study of those is an alternative hypothesis, the MOND, a modification of Newtonian gravitational dynamics suggested by M. Milgrom in 1980s[7, 8, 9], and other models further developed [10, 11, 12]. Although the MOND is out of the mainstream, it accords well with the flat rotation curves of stars in galaxies without the hypothesis of dark matter. According to MOND, when the acceleration is quite weak, Newton's gravitational force is reduced as the single inverse of the distance rather than as inverse square. This is an equivalent to increasing the action of the gravitational force, which has the same effect of the dark matter hypothesis.

According to the latest report, there exists "a correlation between the radial acceleration traced by rotation curves and that predicted by the observed distribution of baryons. The same relation is followed by 2693 points in 153 galaxies with very different morphologies, masses, sizes, and gas fractions. The correlation persists even when dark matter dominates" [13]. Obviously, it is the result of the support for MOND rather than dark matter.

However, until now, no one has sufficiently proven this idea of MOND in theory, and no evidence has been brought forth to falsify the Newton's law of gravitation. Also, the MOND is not able to fit galaxy cluster data without invoking a significant amount of dark matter, so explaining the rotation curves of galaxies without dark matter is not sufficient reason to believe in MOND.

In this study, we suggest a model of celestial body track and a theory of relativistic astrodynamics based on the Newtonian dynamics complemented by special relativity. We shall apply this theory to explain dark matter phenomena and the flat rotation curves in the outer parts of galaxies.

In addition, the phenomena of suspected dark matter have been inferred not only from galaxy rotational curves but also from gravitational lensing. For further argumentation in order to disprove the existence of dark matter halos, in this work, we present a new formula and its theoretical derivation for the angle of deflection of bending light by the gravitational lensing of galaxies, which shows a flat curve of deflection angles in the outer parts of the galaxy.

Furthermore, it is also worth mentioning that the corollaries of the results in this paper will lead to the different explanations of Hubble's redshift other than the effects of Doppler redshift by receding those distant galaxies.[14, 15]

## 1 The relativistic astrodynamics of spiral tracks

If there is a test particle $m$ in outer regions of a galaxy, according to Newton's law of gravity, the centripetal force takes the form

$$F = -G\frac{M_0 m_G}{r^2}, \tag{1}$$

where $r$ is the distance from the particle to the center of the galaxy; $M_0$ is the overall mass of the matters contained within radius $r$, if the test particle is in regions of the outskirt of the galaxy, the $M_0$ is almost the overall mass of the matters of the galaxy; $m_G$ is the gravitational mass of the test particle; $G$ is the gravitational constant. According to Newton's law of motion, the centrifugal force is

$$F = m_I\frac{v_c^2}{r}, \tag{2}$$

where, $v_c$ is the orbital circular velocity relative to the center of the galaxy, $m_I$ is the inertia mass of the particle $m$.

For our problem, the angular momentum of the particle $m$ to the origin of coordinate in the $CM$ system is given by

$$\vec{L} = \vec{r} \times m_I\vec{v}_t, \tag{3}$$

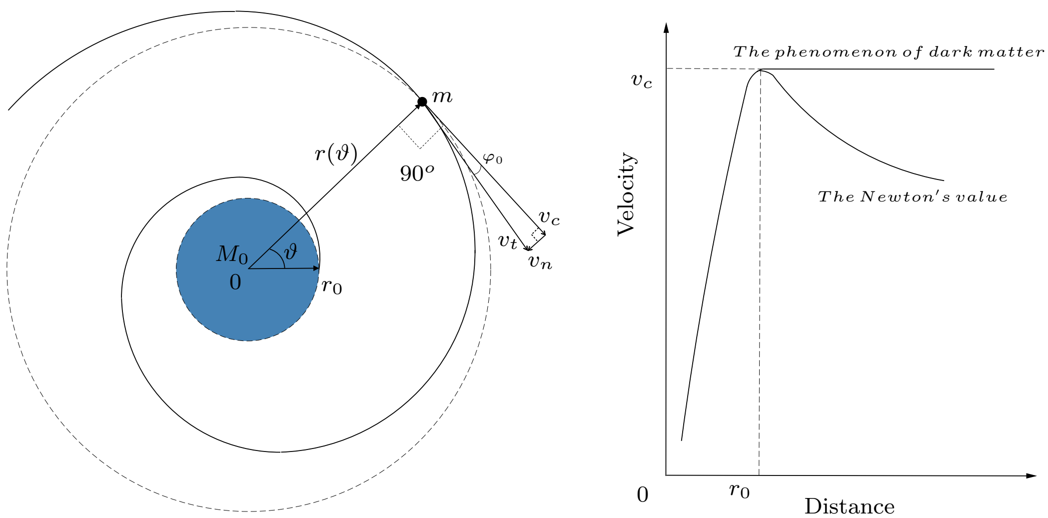
where $\vec{v_t}$ is the orbital tangential speed of the particle $m$.

In terms of our two body self-gravitating system, no matter where the particle $m$ is placed, as there is no torque acting on the particle $m$ about an axis at the center $O$, so it must keep the conservation of angular momentum relative to the center $O$, that is

$$L = m_I\, r\, v_c = constant. \tag{4}$$

Combining equation (4), we rewrite equation (2) by

$$F = m_I \frac{v_c^2}{r} = \frac{L^2}{m_I r^3}, \tag{5}$$




(a) (b)

图 1: The two body gravitational model: (a) The orbit of particle $m$ is a logarithmic spiral track, all the velocities $v_t$, $v_c$ and $v_n$ are constant regardless of its position along the curve. (b) The rotation curve of any galaxy.

Fig.1 shows such a two body system. For setting up a dynamic model of this two-body gravitational system, we take $O$ as the origin of the planar polar coordinate, the orbital equation of motion of particle $m$ is given by

$$\begin{aligned} m_I(\ddot{r} - r\dot{\theta}^2) &= F(r); \\ m_I r^2 \dot{\theta} &= L. \end{aligned} \tag{6}$$

For the convenience of solving differential equation (6), let us set $\mu = 1/r$, we have

$$\dot{\theta} = \frac{L\mu^2}{m_I}; \tag{7}$$

and

$$\begin{aligned}\dot{r} &= \frac{dr}{dt} = \frac{dr}{d\theta}\cdot\frac{d\theta}{dt} = \frac{d}{d\theta}(\frac{1}{\mu})\frac{d\theta}{dt} = -\frac{1}{\mu^2}\frac{d\mu}{d\theta}\dot{\theta} \\ &= -\frac{L}{m_I}\frac{d\mu}{d\theta};\end{aligned} \tag{8}$$

$$\ddot{r} = \frac{d\dot{r}}{dt} = \frac{d}{dt}(-\frac{L}{m_I}\frac{d\mu}{d\theta}) = \frac{d}{d\theta}(-\frac{L}{m_I}\frac{d\mu}{d\theta})\dot{\theta} = -\frac{L^2\mu^2}{m_I^2}\frac{d^2\mu}{d\theta^2}. \tag{9}$$

Substituting (7) and (9) into (6), the orbit differential equation of particle $m$ becomes

$$-\frac{L^2\mu^2}{m_I}(\frac{d^2\mu}{d\theta^2} + \mu) = F(r). \tag{10}$$

By formulas (1) and (5), we get

$$G\frac{M_0 m_G}{r^2} = \frac{m_I v_c^2}{r} = \frac{L^2}{m_I r^3}. \tag{11}$$

However, the force of formula (11) is only a role of action at a distance. For our case, after all, the particle $m$ is running around the $M_0$ in near circular orbit with speed $v_c$, whereupon the finite velocity of gravitational field gives an aberration effect of relativity movement between $M_0$ and the test particle $m$. Fig.2(a) shows the relativistic shift of gravity, if the gravitational field travels at the speed of light, in the point of view of the particle $m$, the relativistic shift of gravity moves the huge gravitational source $M_0$, from the vertical position $O'$ to the side position $O$ by a forward angle $\varphi_0$. A similar example is, in a rainy day without wind, the raindrops do not vertically fall straight but slant down from the front of a runner. It seems as if the raindrops have an oncoming horizontal component velocity, the faster the runner runs, the more the raindrops tilt, and the raindrops also appear to be denser.

The relativistic shift of gravity is similar to the runner's encounter in the rain. In our particular case, the aberration effect is constant because of the circular orbit. For deriving the relationship between velocity $v_c$ and the pitch angle $\varphi_0$, let's say there's a gravitational field $E'$ in the zenith direction as shown in the figure 2(b): if we relate the field intensity to the field lines, from the point of view of the observer $m$ which is at rest in the field, the field lines come from the vertical zenith direction and the field intensity is just the static field $E'$; however, if the observer $m$ is moving with transverse velocity $v_c$, not only the field lines are leaning forward at the moving direction, but also the density of the field lines increases−the field intensity increases from $E'$ to $E$. This is because the field lines are coming down at the speed $c$ and the observer $m$ is moving sidewise at the speed $v_c$, when the vertical distance that went down by the field lines is $ct$, the horizontal distance travelled by $m$ is $vt$, so it is easy to understand that when $m$ horizontally crosses these field lines, there is a doppler effect that compresses the density of the field line by a factor $(1 + v_c/c)$. But that's not the right strengthen factor of the field intensity, because of the contraction effect of ruler in special relativity, the horizontal distance travelled by $m$ should be measured by a shortened ruler, thus, the relativistic correction factor for Newton's gravitational field appears to be

$$\kappa_1 = \frac{1 + v_c/c}{\sqrt{1 - v_c^2/c^2}}, \tag{12}$$

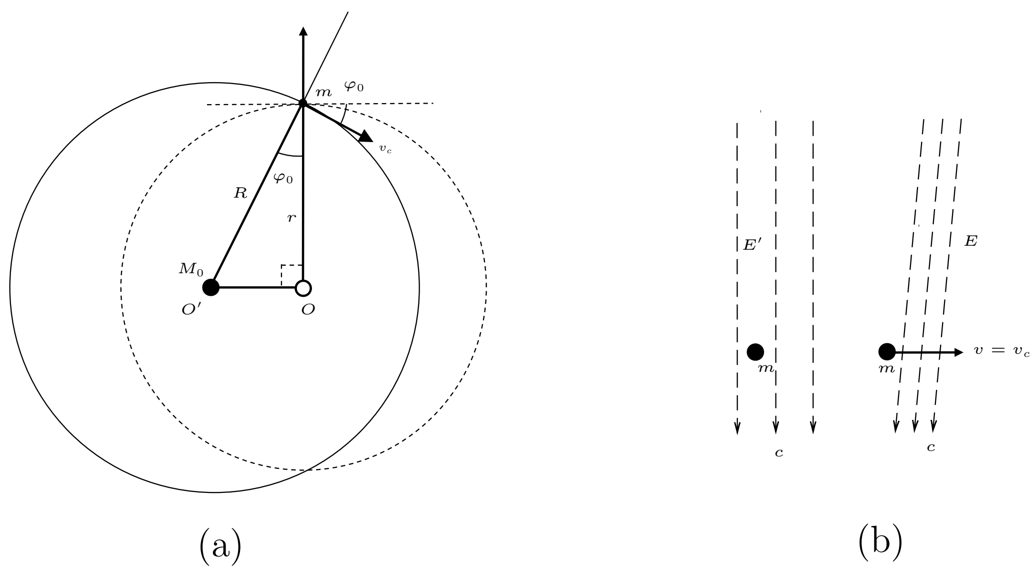



图 2: The finite velocity of gravitational field gives an aberration effect of relativity: (a) The relativistic shift of gravity. (b) The aberration effect of gravity.

however this is just a space transformation factor. In the relativistic conception of time and space, it is necessary to consider the equivalent time conversion factor which is the same as formula (12), therefore the correct relativistic correction factor should be

$$\kappa = (\frac{1 + v_c/c}{\sqrt{1 - v_c^2/c^2}})^2 = \frac{1 + v_c/c}{1 - v_c/c} = \frac{1 + \xi}{1 - \xi}, \tag{13}$$

where, $c$ is the speed of light and $\xi = v_c/c$. As a typical velocity of stars in the Milky Way galaxy, taking $v_c \approx 200kms^{-1}$, then $\xi = v_c/c \approx 10^{-4}$, and $\xi^2 \approx 10^{-7}$, so the formulas (13) can be expanded in a power series. For the series converges rapidly when $\xi = v_c/c$ is small, the terms after the first two are negligible. We therefore have

$$\kappa \approx 1 + 2\xi + 2\xi^2 + 2\xi^3 + \xi^4 \approx 1 + 2\xi. \tag{14}$$

Thus we modified the formula (11) by a reinforced formula

$$F = -\kappa G \frac{M_0 m_G}{r^2} = -\kappa \frac{L^2}{m_I r^3}, \tag{15}$$

that is

$$F \approx -(1 + 2\xi) \frac{L^2 \mu^3}{m_I}. \tag{16}$$

For the (16), we are only taking the first order approximation, which would have to do with dark matter problem. However, in a separate article[14] we proved that, the reinforcing effect of the second-order correction is just the cause of the cosmological redshift and the dark energy problem.

By substituting (16) into (10), we obtain the orbital equation of motion

$$\frac{d^2\mu}{d\theta^2} - 2\xi\mu = 0, \tag{17}$$

which has a solution

$$\mu = c_1 e^{-\sqrt{2\xi}\,\theta}, \tag{18}$$

therefore, the orbit of the test particle $m$ can be given by

$$r = \frac{1}{\mu} = r_0 e^{\sqrt{2\xi}\,\theta}, \tag{19}$$

where, both $c_1$ and $r_0$ are constant.

Obviously, formula (19) is an equation of spiral track. These results suggest that the relativistic corrections create neither a circular orbit nor a precessing elliptical orbit, but a spiral track orbit in the area far away from the center of the galaxy, in where, as the two-body gravitational system, the conservation of angular momentum relative to the center $O$ is strictly true.

## 2 Speed of a moving mass along the logarithmic spiral path

If the vast majority of the mass of a galaxy is located at the neighboring area of the core of the galaxy, the $M_0$ is almost a constant with the distance increasing from the core of the galaxy. In such a self-gravitating system, the equation (11) can be rewritten as

$$v_c = G \frac{M_0 m_G}{v_c r\, m_I} = G \frac{M_0 m_G}{L} = constant \quad (r \geqslant r_0). \tag{20}$$

This result states that, in equation (19), $\xi = \xi_0 = v_c/c = constant$. That is in the area far away from the center of the galaxy, the trajectory of the particle $m$ is the logarithmic spiral track, i.e., the equiangular spiral track. Interestingly, the equiangular spiral track has the fundamental properties, where

$$\varphi_0 = constant; v_t = constant; v_n = constant \qquad (r \geqslant r_0), \tag{21}$$

here, $\varphi_0$ is the pitch angle; $v_t$ and $v_n$ are the orbital tangential velocity and the normal velocity relative to the center of the galaxy accordingly. Thus, the equation (19) becomes

$$r = r_0 e^{\sqrt{2\xi_0}\,\theta} \qquad (r \geqslant r_0), \tag{22}$$

with a constant pitch angle

$$\varphi_0 = \tan^{-1}\sqrt{2\xi_0} \qquad (r \geqslant r_0). \tag{23}$$

Now we demonstrate that, if a particle is moving along the path of logarithmic spiral orbit, not only the velocity of the particle is constant of the motion, which does not vary with time along the logarithmic spiral orbit, but also a constant in the sense of being independent of radius. As in Fig.1(a), if the orbit is a logarithmic spiral track, it must have $\varphi = \varphi_0 = constant$, wherever $m$ is moving in its orbit, there is $v_t^2 = v_c^2 + v_n^2$, and these velocities are all constant regardless of its position along the curve.

However, in any local regions, such as in solar system, the gravitational force coming from the sun is certainly not the only force in such local system, there is stronger non-uniform gravitational field from the center of the galaxy as well, therefore the angular momentum of a planet around the sun is not exactly conserved in strict sense. In short, in any local regions, there are no such conditions of the spatially homogeneity and isotropy as in outer regions of the galaxy. Such limiting conditions should make the velocities of the planets fail to comply with the law of equations (20) in solar system.

By formulas (4) and (20), we can obtain the proportional formula which is

$$m_I\, r = \frac{L}{v_c} = constant \qquad (r \geqslant r_0). \tag{24}$$

Since the formula (22) is the logarithmic spiral track, this interesting result can also be derived from the features of this particular trajectory. By defining $s$ as the arc length of the curve (22), and combining equation (7) and (22), there are

$$\begin{aligned} ds^2 &= dr^2 + (r d\theta)^2; \\ dr &= \sqrt{2\xi_0}\, r d\theta; \\ \frac{d\theta}{dt} &= \frac{L}{r^2 m_I}, \end{aligned} \tag{25}$$

then, we obtain the equations as follows

$$\begin{aligned} ds^2 = dr^2 + (rd\theta)^2 = (2\xi_0 + 1)r^2 d\theta^2; \\ \frac{ds}{d\theta} = \sqrt{(2\xi_0 + 1)}\, r, \end{aligned} \tag{26}$$

and

$$v_t = \frac{ds}{dt} = \frac{L}{m_I r}\sqrt{2\xi_0 + 1} = constant. \tag{27}$$

## 3 The mass of the Milky Way galaxy

Since $r_0$ and $M_0$ are constant, hence the rotation speed $v_c$ does not decrease as the increase of the distance from the critical distance $r_0$. This will lead to a flat rotation curve as if the presence of abundant dark matter in the outer parts of the galaxy. This is a simple solution to the cosmic dark matter problem without the hypothesis of exotic dark matter.

As coincidental for our Milky Way galaxy, the Sun is located close to the turning point of the flat rotation curve. It may approximately take the critical distance $r_0$ at the Sun's location, where[16]

$$r_0 \approx r_\odot \approx 8.29 \pm 0.16kpc \approx 2.6 \times 10^{17}km;$$

$$v_c = v_\odot \approx 239 \pm 5kms^{-1}.$$

Now we can weigh the mass of our Milky Way galaxy

$$M_0 = \frac{r_0}{G} v_c^2. \tag{28}$$

The equation (28) may accurately determine the mass of our Milky Way galaxy without the dark matter. If letting $r_0 \approx 2.6 \times 10^{17}km$, and the rotation speed $v_c = v_0 = v_\odot \approx 240kms^{-1}$, we obtain the virial mass of the Milky Way's matter

$$M_0 \approx 2.2 \times 10^{41}kg \approx 1.1 \times 10^{11}M_\odot,$$

here, $M_\odot \approx 2.0 \times 10^{30}kg$.

## 4 The pitch angle of logarithmic spiral orbit in the outer regions of the Milky Way galaxy

In regions of $r \geqslant r_0$, the orbit formula of the test particle is given by

$$r = r_0 e^{\sqrt{2\xi_0}\,\theta} = r_0 e^{(\sqrt[4]{4GM_0/r_0c^2})\cdot\theta}, \tag{29}$$

where

$$\xi_0 = \frac{v_c}{c} = \sqrt{\frac{GM_0}{r_0 c^2}}. \tag{30}$$

For our Milky Way galaxy, the pitch angle of logarithmic spiral orbit is

$$\varphi_0 = \tan^{-1}\sqrt{2\xi_0} = \tan^{-1}\sqrt{\frac{2v_c}{c}} = \tan^{-1}[\frac{4GM_0}{r_0c^2}]^{1/4} \approx 2.3^0, \tag{31}$$

where, $G \approx 6.7\times10^{-11}m^3kg^{-1}s^{-2}$; $M_0 \approx 2.2\times10^{41}kg$; $r_0 \approx 2.6\times10^{20}m$; $c = 3\times10^8ms^{-1}$. Therefore, the Sun's normal velocity towards the galaxy center would be

$$v_n = v_c \times \tan\varphi_0 \approx 9.6kms^{-1}, \tag{32}$$

where $v_c = v_\odot \approx 240kms^{-1}$.

This prediction can be confirmed by astronomical observations. The study based on observational data has shown the orbital normal velocity of the sun (radially inwards w.r.t. the local standard of rest) is[17]

$$v_n \approx 10 \pm 0.36kms^{-1}.$$

The rotation curve of the Milky Way galaxy without the dark matter is shown in Fig.3

Note that, since the arms of spiral galaxies are generally logarithmic spiral in form, there should be no confusion that the pitch angle $\varphi_0$ never refers to the pitch angle of spiral arm of the Milky Way, which almost looks just like the logarithmic spiral shape. A recent research estimated the pitch angle of the Local arm to be $10.1^0 \pm 2.7^0$[18]. Therefore, it is apparent that stars just pass through the arms as they travel in their own quasi logarithmic spiral orbits, but it is not clear why these stars interspersed in certain areas to give the apparent scene of logarithmic spiral arms.

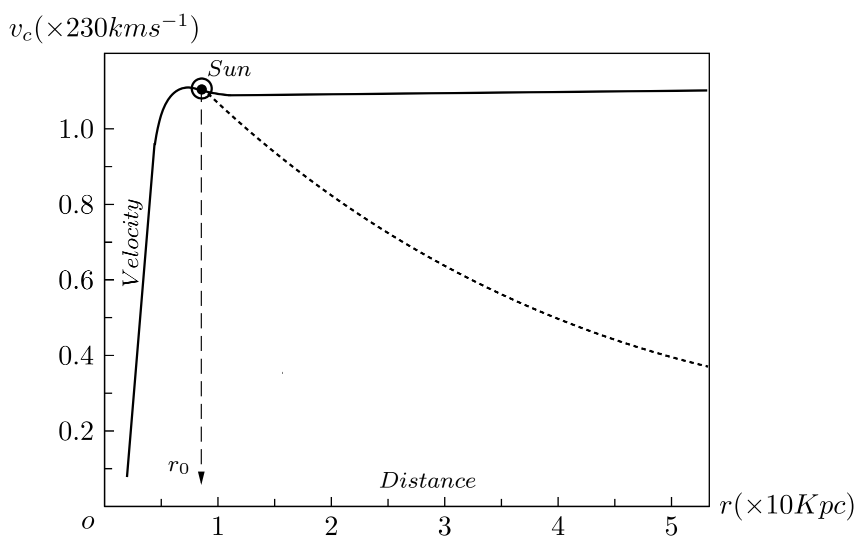
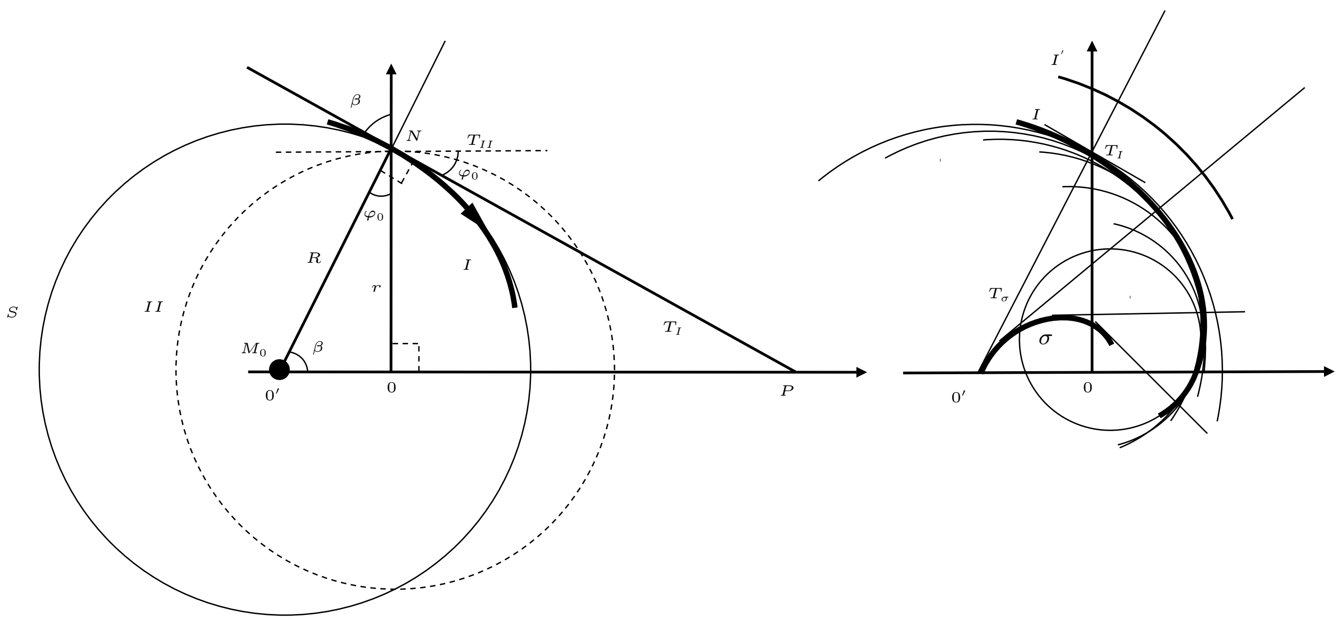



图 3: Rotation curves of the Milky Way galaxy: the solid-line is predicted by the relativistic astrodynamics and has been observed; the dotted line is expected by Newton's theory.

# 5 Geometric and physical significance of equiangular spiral track

We have shown that the aberration effect of gravitational field spread is the cause of the equiangular spiral track, which resulted in the enduring puzzles of dark matter. The results of these relativistic gravity can also be deduced from geometric idea.

Based on the Newton's equation (11), there is a circular orbit $S$ as shown in Fig.4(a). However, the circular orbit is not stable, the circular orbit $S$ of a mass moving at constant speed would be converted to an equiangular spiral track $I$ by the aberration effect of gravity, which is the solution of orbital equation of motion (17). In geometry, as shown in Fig.4(a), this solution is equivalence to a proposition that the center $O^{'}$ of gravity of point $N$ in curve $I$ would be moved to $O$ by aberration shift, as if the center of $M_0$ is not at $O^{'}$, but rather $O$, where $O^{'}$ is the center of curvature circle of point $N$ in curve $I$, and $O$ is a fixed point, the origin of polar coordinate. Now we shall prove the following result: if the curve $I$ is equiangular spiral, the proposition must be true.




(a) (b)

图 4: The geometric interpretations of forming the equiangular spiral orbits and their physical and mathematics meanings: (a) The relativistic shift of gravity changes the orbital pattern of stars. (b) The natures of the equiangular spiral orbits.

Let us suppose curve $I$ is the equiangular spiral (22), by taking the derivatives of the equation of (22), that is $dr/d\theta = r'_\theta = \sqrt{2\xi_0}\cdot r\,; dr^2/d\theta^2 = r''_\theta = 2\xi_0 r$ , then plug them into the formula of curvature radius, we get the curvature radius of point $N$ in curve $I$

$$R = \frac{(r^2 + r'^2_\theta)^{\frac{3}{2}}}{r^2 + 2r'^2_\theta - rr''^2_\theta} = r\sqrt{1+2\xi_0}, \tag{33}$$

where $\sqrt{2\xi_0} = \tan\varphi_0 = \frac{\overline{OO'}}{\overline{ON}}$, hence the curvature radius of point $N$ can be written as

$$R = \frac{r}{\cos\varphi_0}, \tag{34}$$

that is just the length of polar normal $\overline{O'N}$. This proves that as long as the $O'$ is the center of curvature circle of point $N$ in the equiangular spiral $I$, the origin of polar coordinate must be $O$. And because of $\xi_0 = v_c/c$; $\varphi_0 = \tan^{-1}\sqrt{2\xi_0}$, hence the result also proves that the $M_0$ seems to move forward with a deflection angle of $\varphi_0$, meaning there is an aberration shift of source of gravity. Obviously there is $R > r$, and $M_0$ remained unchanged, which means that the aberration shift of gravitational field spread is reinforcing the gravity, i.e.

$$\frac{F(r)}{F(R)} = \frac{R^2}{r^2} = (\frac{1}{\cos\varphi_0})^2 = 1 + 2\xi_0. \tag{35}$$

This is the geometric and physical significance of reinforcing formula (16), especially on a galactic scale, such reinforcing effect changes the orbital pattern of stars.

In the following sections, we will take a closer look at the physical and mathematics meanings of equiangular spiral orbit.

**i**) The equiangular spiral has the property that the pitch angle $\varphi_0$ at any point on the curve is constant, as shown in Fig.1(a) or Fig.4(a). This nature makes the equiangular spiral closely related to the circle (in case of a circle, the pitch angle $\varphi_0 = 0$).

For the equiangular spiral (22), if $\sqrt{2\xi_0} = 0$, then $\varphi_0 = \tan^{-1}\sqrt{2\xi_0} = 0$, the equiangular spiral $I$ degrades into the dotted circle $II$ as shown in Fig.4(a), moreover the circle $II$ will overlap the circle $S$. Obviously, in a local area, the circle $II$ is the approximate limit of equiangular spiral $I$, and at the weak field and low speed approximation($\xi_0 = v_c/c \approx 0$; the pitch angle $\varphi_0 = \tan^{-1}\sqrt{2\xi_0} \approx 0$ ), the circle $S$ is the approximate limit of both $I$ and $II$.

**ii**) The equiangular spiral $I$ is an envelope curve of a family of circle $S$, as shown in Fig.4(b), and the trajectory $\sigma$ composed of the centres of these curvature circles is also an equiangular spiral with the same equation as $I$. The equiangular spiral $\sigma$ forms the evolute of equiangular spiral $I$, the only difference is that there is an angle of rotating about the origin of polar coordinate. Of course, the original equiangular spiral $I$ is the involute of the curve $\sigma$, the $I$ and $\sigma$ have vertical tangent $T_I$ and $T_\sigma$ at corresponding point, that way, the evolute $\sigma$ has been formed by the envelope of a family of normal $T_\sigma$.

In the problem of galactic rotation curves, based on these properties of the equiangular spiral, we can infer that the trajectory of the gravity center of a galaxy is also an equiangular spiral, which is the evolute of trajectories of the celestial bodies in outer regions of the galaxy and has the same orbital equation as these celestial bodies.

**iii**) One equiangular spiral evolute $\sigma$ can have many of the same equiangular spiral involute $I$、$I^{'}\cdots$, as shown in Fig.4(b). This means that, if the trajectory of the gravity center of a galaxy is an equiangular spiral, then all the trajectories of the celestial bodies in outer regions of the galaxy are the same equiangular spiral and must have the same speed—there appears to be a “logic-defying” flat rotation curve.

**iv**) In an equiangular spiral, there exists limited arc length and countless circles from any point to the origin of polar coordinate. So, if a point moves along the curve to the origin of polar coordinate, it will never get there. As shown in Fig.4(a), the arc length of the equiangular spiral $I$ from point $N$ to $O$ is equal to $\overline{NP}$, however, it requires to be countless circles for reaching the point $O$.

Corresponding to the galactic model, in this case, any celestial body in galaxy could get the centre only after infinite cycles and time, it means any content of the universe is impossible to get there, and nothing is in the singularity.

# 6 The relativity of inertia in gravitational field

In the equation (4), by the law of conservation of angular momentum, there must be $m_I = L/(v_c \cdot r)$; and in the equation (27), by the fundamental properties of logarithmic spiral track, it must have $r \cdot m_I = constant$. Both the equations (4) and (27) get the same result

$$m_I \propto \frac{1}{r} \qquad (r \geqslant r_0). \tag{36}$$

This result means that, in the galaxy's gravitational field, the effective inertia $m_I$ of an object is inversely proportional to $r$ in the regions of spiral arms and halo of galaxies, where the flat rotation curve appears.

We can define a critical distance $r_0$ as the critical radius to the center of a galaxy, from where a flat rotation curve began to appear. At this critical radius, for an object there is $m_I = m_G$, and in the regions where $r \geqslant r_0$, by the law of the equation(4), we have

$$m_I v_c r = m_G v_c r_0 = constant. \tag{37}$$

Thus we acquire an expression of the relativity of inertia in gravitational field of a galaxy

$$\frac{m_I}{m_G} = \frac{r_0}{r}. \tag{38}$$

The ratio of the inertia to the gravitational mass of a particle begins 'running' in the part of the logarithmic spiral tracks. But this, apparently, if its inertial mass $m_I$ is inversely proportional to $r$ in the regions $r \geqslant r_0$ of the galaxy while the gravitational mass $m_G$ is invariable, the observed flat rotation curve can be explained−it will naturally move faster. After all, **the gravitational mass $m_G$ of the particle is independent of $r$ in the equation (1), otherwise the Newton's law of gravitation would be meaningless**.

This also means that there are different ways of keeping the conservation of angular momentum and the energy conservation. In the area of the logarithmic spiral track, the kinetic energy of the particle is increased by increasing inertia mass instead of by increasing its speed. Hence, the relationship of would imply that a particle becomes more massive when it moves from the halo to the core of the galaxy within the regions of the logarithmic spiral track.

Where does the extra kinetic energy come from? In reality, the extra kinetic energy of the particle comes from the gravitational potential energy of the system. When the particle $m$ moves toward to $M_0$ along a logarithmic spiral orbit, the gravitational potential energy can be converted into the kinetic energy, while the gravitational field itself keeps more and more negative potential energy. This relationship between the kinetic energy and the potential energy abides by the Clausius virial theorem, and the law of energy transform and conservation, that is

$$2T + W = 0 \tag{39}$$

where, $T$ is the kinetic energy of the moving partical and $W$ is the gravitational potential energy of two body system. Therefore, in numerical value there is

$$\frac{1}{2}G\frac{M_0 m_G}{r} = \frac{1}{2}m_I v_t^2, \tag{40}$$

for $v_t = constant$, we can also get the same inverse relation as the equation (36). Obviously, these results are in accord with Einstein's use of the Mach's principle: "**· · · inertia originates in a kind of interaction between bodies· · ·** ".[19]

## 7 The Milgrom's law in theory

According to MOND, in weak gravitational field of outer regions of a galaxy, there exists a critical value of acceleration[20] which is $a_0 \approx 1.2 \times 10^{-10} ms^{-2}$, and when $a < a_0$, the gravity will be beyond the expectation of Newton's theory, that is

$$a \propto \frac{1}{r^2},\ (a > a_0); \qquad a \propto \frac{1}{r}, (a < a_0). \tag{41}$$

However by Newton's law, that is

$$m_I a = G\frac{M_0 m_G}{r^2}, \tag{42}$$

if there is $r_0$ corresponding to $a_0$, where $r \geqslant r_0$, $a \leqslant a_0$, we get

$$a = G\frac{M_0 m_G}{m_I r^2} \propto \frac{1}{r}, \qquad (r \geqslant r_0), \tag{43}$$

here, by (24) or (27), $m_I r = constant$.

Thus the Modified Newtonian dynamics (MOND) can be established on the principle of relativity of inertia, and the Milgrom's law is proved to be true in theory.

# 8 The Pioneer anomaly and the dark matter phenomenon at the outside edge of the solar system

## 8.1 Localized equivalence principle

However, if one were to apply the equations (20) or (22) equally well to the solar system, it would result in absolutely enormous deviation from the equivalence principle in solar system. Certainly, it is contrary to observations.

Actually, in a local system such as our solar system, since its scale is much smaller than that of the galaxy, the running of $m_G/m_I$ is small. In principle, along the radius direction of galactic plane, in theory, we can take the derivative of the equation (38) with respect to $r$, then we get

$$\triangle\, m_I = -\frac{m_G r_0}{r^2} \cdot \triangle r. \tag{44}$$

The relative change rate of inertia mass is

$$\frac{\triangle m_I}{m_G} = -\frac{r_0}{r^2} \cdot \triangle r. \tag{45}$$

This result shows that, in a small enough space of a galaxy, where $\triangle r \approx 0$, by the equation (44), there will be $\triangle m_I \approx 0$ and $m_I \approx m_G$, the equivalence principle remains valid, so that was called the local equivalence principle.

As one of fundamental hypotheses of general relativity, the equivalence principle was strictly established only in local inertial system of coordinate or in the Galilean regions defined by Einstein. However, in the global sense of universe, although the ratio of the inertia mass to the gravitational mass of an object still is a constant in a local fixed area, their ratio is no longer a universal constant in the view of different inertial observers. Certainly, this does not mean the equivalence principle is incorrect and contradicts the fundamental tenet of the general relativity that refers to the locally measured inertia mass and gravitational mass of an object. It is just the further evidence that there is only the localized equivalence principle. Actually, the deviation from the localized equivalence principle is the result of the bending space-time, in other word, the effect is caused by non-uniform gravitational field and the relative acceleration between two local Lorentz frames.

### 8.2 The relativity of inertia of multi-body systems

Divided by the $c^2$ on both sides of the equation (40), there is

$$\frac{1}{2}G\frac{M_0 m_G}{rc^2} = \frac{1}{2}m_I\frac{v_t^2}{c^2}. \tag{46}$$

According to Einstein's mass-energy relationship, the left side of the equation (46) can be interpreted as "the mass of the gravitational field" which associates with potential energy of the two-body system and is proportional to the kinetic energy of the moving body, the more inward the object moves along a logarithmic spiral orbit, the more gravitational energy will be released. The equations (40) and (46) show that, **the gravitational energy absorbed by the moving body must be transformed to its kinetic energy, however its speed is not changing, the increase of the inertial mass is the only way to enhance kinetic energy**.

Since gravitational energy and inertial mass are scalars, we could easily model our relativity of inertia of multi-body systems, the equation (38) can be promoted to

$$m_I = \frac{m_G}{1-\Lambda}, \tag{47}$$

and

$$\Lambda = \frac{\triangle\sum GM_i/R_ic^2}{\sum GM_i/R_ic^2} \tag{48}$$

where $m_G$ is the gravitational mass of a moving body, and $m_I$ is its inertial mass; $\Lambda$ is the relative increment of the inertia mass; $M_i$ are all the observed gravitational masses in the universe: planets and stars even those distant galaxies and galaxy clusters; $R_i$ are defined as the coordinate distances from the moving body to the centres of these gravitational masses.

The equations (47) and (48) are useful, accurate and effective formulas which are the physical and mathematic expression of Mach's principle. As a case in point, there is a much simpler way to solve the puzzle of Pioneer anomaly by using these equations.

### 8.3 The Pioneer anomaly

The Pioneer 10 and Pioneer 11 were launched in 1972 and 1972 respectively. Over the years, from the JPL's analysis, there is an anomalous acceleration $a_p$ towards the Sun[21]: for Pioneer 10, $a_p \approx (8.09\pm0.20)\times10^{-10}ms^{-2}$ , and for Pioneer 11, $a_p \approx (8.56\pm0.15)\times10^{-10}ms^{-2}$.

There are significant similarities between Pioneer anomaly and the phenomena of dark matter. Specifically, the problem is very related to the Modified Newtonian Dynamics (MOND or Milgrom's law)[22]. Here we show that the anomalous acceleration acted on the spacecraft is due to deviation from the equivalence principle and the relativity of inertia.

When the spacecraft is far away from the center of solar system, the largely relative increment of the gravitational potential is only from the Sun, thus the formula (48) can be simplified as

$$\Lambda = \frac{\triangle \sum GM_i/R_i c^2}{\sum GM_i/R_i c^2} \approx \frac{\triangle GM_s/rc^2}{GM_0/R_g c^2 + GM_s/R_s c^2 + GM_e/R_e c^2}, \tag{49}$$

where $r$ is the distance between the Sun and the spacecraft; the gravitational constant $G \approx 6.67 \times 10^{-11} m^3 kg^{-1} s^{-2}$; $M_s$ is the mass of the Sun and $M_s \approx 2.0 \times 10^{30} kg$; $R_s$ is the distance between the Earth and the Sun and $R_s \approx 1AU \approx 1.5 \times 10^{11} m$; $M_e$ is the mass of the Earth and $M_e \approx 6.0 \times 10^{24} kg$; $R_e$ is the radius of the Earth and $R_e \approx 6.4 \times 10^6 m$; $M_0$ is the mass of the Milky Way and $M_0 \approx 2.2 \times 10^{41} kg$; $R_g$ is the distance from the Sun to the center of the Milky Way and $R_g \approx 2.6 \times 10^{20} m$.

In theory, let's review the properties of the equations (49). The formula (49) can be expanded in a power series at $r = r_i$, that is

$$\begin{aligned} \Lambda &\approx \frac{-GM_s/r_i^2 c^2}{GM_0/R_g c^2 + GM_s/R_s c^2 + GM_e/R_e c^2}(r - r_i) \\ &= \frac{-M_s/r_i^2}{M_0/R_g + M_s/R_s + M_e/R_e}(r - r_i) \\ &= -\Lambda_0 (r - r_i), \end{aligned} \tag{50}$$

where the derivative function

$$\Lambda_0 = \frac{M_s/r_i^2}{M_0/R_g + M_s/R_s + M_e/R_e}. \tag{51}$$

As the most simple case of the formula (51), in two-body system, there is $\Lambda_0 = 1/r_i = 1/r_0$, thus the formula (47) can be changed back to formula (38) that shows the Milgrom's phenomena.

Taking the Sun as the origin of the relative coordinate and substituting (51) and (50) into (47), we get the Taylor power series of (47), that is

$$\begin{aligned} m_I &= \frac{m_G}{1 - \Lambda} \approx m_G [1 + \Lambda + \Lambda^2 + \cdots\cdots] \\ &= m_G [1 - \Lambda_0 (r - r_i) - \Lambda_0^2 (r - r_i)^2 - \cdots\cdots]. \end{aligned} \tag{52}$$

Substituting this expression of inertial mass into the dynamic equation of gravity, we get the acceleration towards the Sun, which is

$$\begin{aligned} a &= \frac{GM_s m_G}{r^2 m_I} = GM_s/r^2 [1 - \Lambda_0 (r - r_i) - \Lambda_0^2 (r - r_i)^2 - \cdots\cdots] \\ &= \frac{GM_s}{r^2} + \frac{GM_s}{r^2} \Lambda_0 (r - r_i) + \frac{GM_s}{r^2} \Lambda_0^2 (r - r_i)^2 + \cdots\cdots, \end{aligned} \tag{53}$$

here, the first term is the result of Newton's gravitational pull; the second one is the Milgrom's term; the third is precisely the anomalous acceleration which tends to be constant as the distance getting farther from the center of the solar system, that is

$$a_p = \frac{GM_s}{r^2} \Lambda_0^2 (r - r_i)^2 \approx GM_s \Lambda_0^2, \qquad r \gg r_i. \tag{54}$$

However, in the equation (54) $\Lambda_0$ is the derivative function of curve (50), it will change along with the distance. Only if the spacecraft gets farther from the center of the solar system and beyond the critical radius $r_c$, the curve (50) starts to flatten and the equation (54) tends to be constant.

The masses of the solar system are 1.0014 Solar masses, which are mainly made up of the Sun itself and the mass of the Jupiter, so it can be assumed that the orbit radius of Jupiter is the critical radius of masses of the solar system. Therefore we may take the aphelion of the Jupiter as the critical radius $r_c \approx 5.5AU(816,520,800km)$. Anyhow it is just a theoretical limit, in effect, based on the data from the JPL's analysis, i.e., $a_p \approx 8.74 \times 10^{-10} ms^{-2}$, by the equation (54) we can obtain $r_c \approx 6.4AU \approx 9.6 \times 10^{11} m$.

In the frame of reference in which the Sun is the origin of the relative coordinate, when the spaceship left earth, there is $r = R_s$ and $m_I = m_G$, then we get

$$a_p = \frac{GM_s}{(r+R_s)^2}[\frac{M_s/(r+R_s)^2}{M_0/R_g + M_s/R_s + M_e/R_e}(r-R_s)]^2, \qquad R_s \leqslant r \leqslant r_c; \tag{55}$$

and

$$a_p \approx GM_s\Lambda_0^2 = GM_s(\frac{M_s/r_c^2}{M_0/R_g + M_s/R_s + M_e/R_e})^2, \qquad r \geqslant r_c. \tag{56}$$

Fig.5 shows the theoretical curve of equations (55) and (56). Obviously, if one were to look at the phenomenon of Fig.5, it would seem to indicate that there was dark matter at the outskirts of our solar system. In fact, the real reason for this anomaly is the additional gravity caused by the relativity of inertia in the gravitational field.

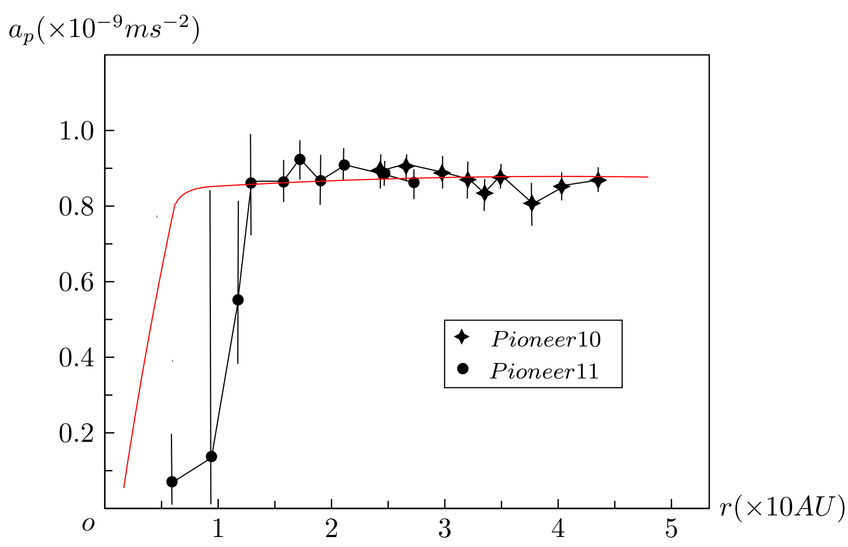



图 5: The data and error bars are the records of Pioneer 10 (1981-1989) and Pioneer 11 (1977-1989)[21]; the red solid line is the theoretical curve of the equations (55) and (56).

Although NASA's scientists had doubted about the mass variation of the spacecraft, they finally still attributed it to other possible factors：*"Once in deep space, all major forces on the spacecraft are gravitational. The Principle of Equivalence holds that the inertial mass* ($m_I$) *and the gravitational mass* ($m_G$) *are equal. This means the mass of the craft should cancel out in the dynamical gravitational equations. As a result, the people who designed early deep-space programs were not as worried as we are about having the correct mass. When non-gravitational forces were modeled, an incorrect mass could be accounted for by modifying other constants. For example, in the solar radiation pressure force the effective sizes of the antenna and the albedo could take care of mass inaccuracies"*(see the references (45) of that article[21]).

# 9 The enhancement effect of gravitational lensing of galaxies

## 9.1 The Einstein lensing

A gravitational lens based on the Einstein equivalence principle is called Einstein lens. In Einstein's theory, the deflection angle of the Einstein lensing is

$$\delta\varphi = \frac{4GM}{rc^2}, \tag{57}$$

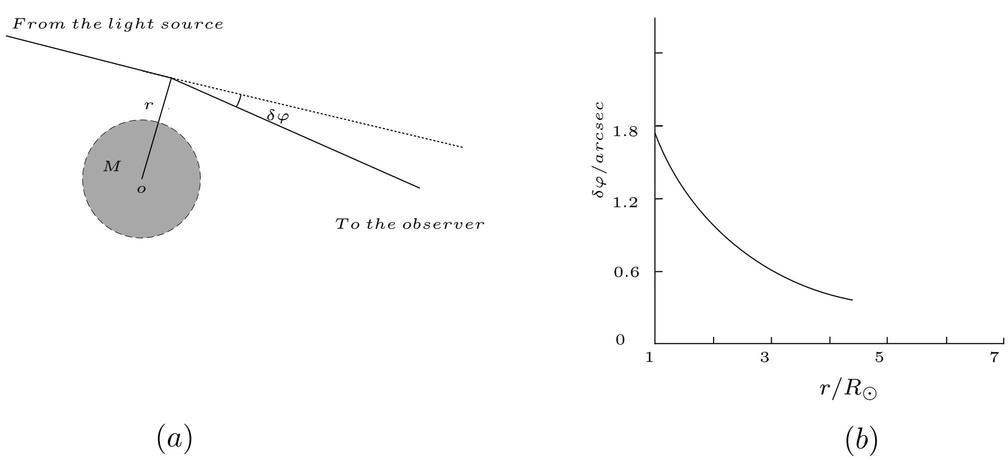



图 6: The Einstein lensing: (a) A gravitational lens based on the Einstein equivalence principle. (b) The Einstein's value.

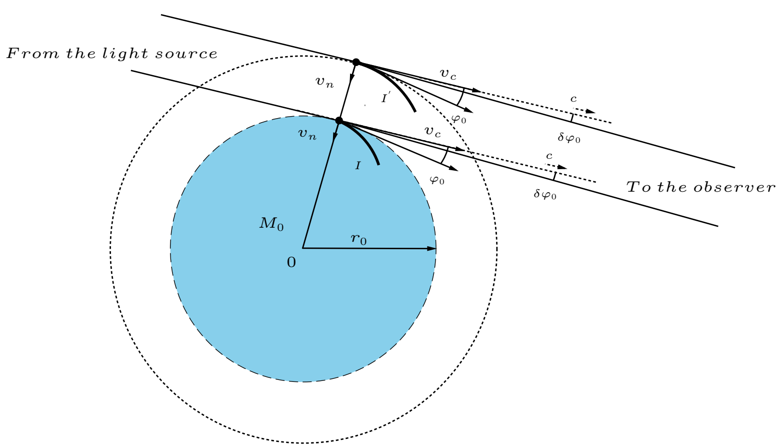
where $G$ is the universal constant of gravitation and $c$ is the speed of light in a vacuum; $r$ is the impact parameter; $M$ is the gravitational mass of the massive celestial body. The physical interpretation of equation (57) is shown in Fig.6(a). In the case of the Sun, the functional relationship between angle of deflection and the impact parameter is shown in Fig.6(b), in where, if the starlight just passes through surface of the sun, there are $r = R_{\odot} \approx 6.96 \times 10^5 km$ and $M = M_{\odot} \approx 1.99 \times 10^{30} kg$, this gives a value of deflection angle

$$\delta\varphi = \frac{4GM_{\odot}}{R_{\odot}c^2} \approx 1.75'', \tag{58}$$

which had been confirmed by observation in May 1919 by Arthur Eddington and Frank Watson Dyson[23]. However, because of the local restrictions of Einstein equivalence principle, formula (57) should only be valid in local area such as the narrow confines of our solar system.

## 9.2 The gravitational lensing of galaxies

If the gravity is not action at a distance, what effect is it having? According to the result in the chapter 5, whether the logarithmic spiral orbit $I$ or the orbit $I'$, as shown in Fig.4(b), all the trajectories of the celestial bodies in outer regions of a galaxy are the same logarithmic spiral and must have the same speed and pitch angle $\varphi_0$, in the sense of being independent of radius.



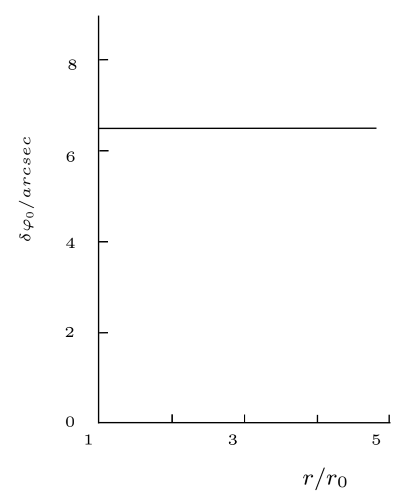




图 7: Gravitational lensing of an isolated galaxy: (a) A gravitational lens of galaxy based on the relativistic astrodynamics. (b) The value of the Milky Way galaxy.

One can only acknowledge that, in the galaxy's gravitational field, the photons have the same normal velocity $v_n$ and the speed of transverse velocity is $v_c = c$, as shown in Fig.7(a), by formula (31), we get the deflection angle of a galaxy lens

$$\delta\varphi_0 \approx \tan\delta\varphi_0 = \frac{v_n}{c} = \frac{v_c}{c} \times \tan\varphi_0 = \sqrt{2}(\frac{GM_0}{r_0 c^2})^{3/4}, \tag{59}$$

where $M_0$ is the overall mass of the matters contained within a critical radius $r_0$. This is obviously a much larger value than the Einstein lensing. Besides that, since $r_0$ and $M_0$ both are constant, hence the deflection angle of $\delta\varphi_0$ does not decrease as the increase of the impact parameter from the critical distance $r_0$. This will lead to a flat curve of $\delta\varphi_0$ as if the presence of abundant dark matter halos in the outer parts of the galaxy.

In the case of our Milky Way galaxy, it may approximately take the critical distance $r_0$ at the Sun's location, where $r_0 \approx 8.29{\pm}0.16kpc \approx 2.6{\times}10^{17}km$; $M_0 \approx 1.1{\times}10^{11}M_\odot \approx 2.2{\times}10^{41}kg$, this gives a value of the deflection angle of our Milky Way galaxy

$$\delta\varphi_0 \approx \tan\delta\varphi_0 = \sqrt{2}(\frac{GM_0}{r_0 c^2})^{3/4} \approx 6.5'', \tag{60}$$

which is shown in Fig.7(b). The case of the Milky Way galaxy shows an example of how to use the formula (59) to weigh the mass of galaxies that would eliminate the need for dark matter halos.

## 10 Conclusions

In this paper, we have suggested a theory of relativistic astrodynamics to explain the flat galaxy rotation curves without the need of dark matter. The result shows that only if stars are far away from the galactic center, Newton's law of gravity and law of motion as well as the theory of relativity would lead to logarithmic spiral tracks and the flat rotation curves.

The study has shown that the flat rotation curve of a galaxy indicates the high homogeneity and isotropy in external galaxies, by which the celestial bodies in outer region of a galaxy can keep conservation of angular momentum strictly, however such condition for a planet around the Sun is not true.

By these theory, the Modified Newtonian dynamics (MOND) can be established on the principle of relativity of inertia.

However, the value of $m_G/m_I$ has always been regarded as a universal constant, not only in the local inertial reference frame but also between distant objects with different reference frame of the viewer, therewith the mass has been simply canceled out from both sides of the equation (11), and the dark matter problem arises.

The theoretical result of a long-range anomalous acceleration acting on Pioneer 10 and 11 is consistent with the data of NASA, and basically ruled out other assumptions for most likely cause such as “gas leaks”, “thermal recoil force”[24] and dark matter.

The principle of relativity of inertia is the physical expression of Mach's principle, which has universal sense. The equations based on this principle can be used for calculation of exact relativity of inertia in the solar system, and help us to more accurately predict the future position and orbits of asteroid closer to the Earth.

We have obtained an equation to determine the relationship between the gravitational lensing effect of the isolated galaxy and its ordinary mass without dark matter halo. The values of the deflection angle of a isolated galaxy may only relate to the mass of the galaxy's core area, and appear to be a flat curve regardless the impact parameter, which is consistent with the flat rotation curve of the galaxy.

However, the Einstein lensing is the local effect of gravitational lens, it is based on the local Einstein equivalence principle and does not take into account of the limitation of the speed of gravity in the global significance. Therefore, the Einstein lensing is valid only in the local district around a star in its galaxy.

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